When the SST Standard Model Underestimates Market Risk

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Abstract

This paper[1] is the report of an experiment. Every year all the companies under the supervision of the *Swiss Federal Office for Private Insurance* (FOPI) deliver the sensitivities of their portfolios to a set of predefined risk factors. Out of these sensitivities it is possible to compute the risk measures for market risk and the resulting capital requirement. Using a linear approximation, these measures can be computed analytically. Given these sensitivities, it is also possible to include the convexity effect using a Monte Carlo simulation.

The results show that the SST Standard Model is unsuitable for *all* the Life companies since it substantially *underestimates* risk.

1 Introduction

All the companies under the supervision of the *Swiss Federal Office for Private Insurance* (FOPI) which participate in the SST Field Test deliver the sensitivities of their portfolios to a set of predefined risk factors. Using these sensitivities it is possible to compute analytically the risk measures (Value at Risk and Conditional Value at Risk) for market risk. This is done automatically by the *SST* Standard Template, a Microsoft Excel file elaborated by FOPI to be filled out by each insurance company (available at: www.bpv.admin.ch).

The SST Standard Model assumes a Gaussian variance-covariance portfolio structure with linear approximation (described in the next section). This simplification is useful if you need a closed form solution

[1]This paper expresses the personal views and opinions of the authors. Please note that the Office the authors work for neither advocate nor endorse the use of the valuation techniques presented here for any purpose (including reporting and risk management).

for VaR and $CVaR$ but leads to a loss of information about the convexity effect. This effect can be more or less pronounced. In order to include the convexity effect we have to drop the linearity assumption. In this case, the risk measures cannot be expressed analytically and other methodologies have to be explored. We investigate two of these methodologies: the Monte Carlo simulation and the Cornish-Fisher approximation.

We proceed as follows. Section 2 shows how to formally obtain the SST Standard Model for market risk. Section 3 shows how to include the convexity effect (an additional assumption is required). Two subsections introduce to the basic ideas about the Monte Carlo method (3.1) and the Cornish-Fisher approximation (3.2). A note on how to obtain numerically the second order Taylor expansion is in the Section 4. The numerical results and comparison between the different lines of business are exposed in Section 5.

We would like to stress the fact that *options and guarantees* embedded in Life insurance contracts are not considered in this analysis because they are not part of the SST for the Field Test 2006.

The results show that for Property and Casualty (P&C) and Health companies there is no advantage in adding the convexity effect to the linear approximation. For all the Life companies the SST Standard Model is *unsuitable* because it *substantially underestimates market risk*.

2 The SST Standard Model for Market Risk

In this section we show how to formalize the SST Standard ModelṪhe experienced reader will recognize the SST Standard Model as a special case of the RiskMetrics^{TM} model. Here we follow the notations of the RiskMetrics^{TM} Technical Document [3]. More details about the SST implementation are available in [4].

Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space, $\mathcal{F}$ a filtration on Ω. $W = \{W_t\}$ denotes a Wiener process on $\mathcal{F}$. Every economic *risk factor* is modeled trough its level value P_t. For P_t we assume the dynamics:

$$dP_t = \mu P_t dt + \sigma P_t dW_t \tag{1}$$

Thus the *log*-returns $r_{t,T} = \log(P_T/P_t)$. can be expressed as:

$$r_{t,T} = (\mu - \frac{1}{2}\sigma^2)(T-t) + \sigma\sqrt{T-t} \cdot \epsilon, \tag{2}$$

where $\epsilon \sim \mathcal{N}(0,1)$. Suppose we have n risk factors and that $\mu = \frac{1}{2}\sigma^2$ so that the previous equation becomes:

$$r_{t,T}^{(i)} = \sigma_i \sqrt{T-t} \cdot \epsilon^{(i)} \text{ with } \epsilon^{(i)} \sim \mathcal{N}(0,1) \tag{3}$$

for $i = 1, \ldots, n$. Thus $r_{t,T}^{(i)} \sim \mathcal{N}(0, \sigma_i \sqrt{T-t})$. The SST assumes $T - t = 1$ year, so that we can neglect the time index and write:

$$r^{(i)} \sim \mathcal{N}(0, \sigma_i),$$

Assuming that the risk factors are not independent, we can denote the correlations as

$$\rho_{ij} := corr(\epsilon_i, \epsilon_j) \tag{4}$$

and with

$$\Sigma_{ij} = cov(r^{(i)}, r^{(j)}) = \sigma_i \sigma_j \rho_{ij}.$$

Thus $\mathbf{r} = [r^{(1)}, \ldots, r^{(n)}]$ is a multivariate normal random variable with expected value $\mathbf{0}$ and covariance matrix $\mathbf{\Sigma} = [\Sigma_{ij}]$:

$$\mathbf{r} \sim \mathcal{N}_n(\mathbf{0}, \mathbf{\Sigma}) \tag{5}$$

Let us denote by V the value of our portfolio. The portfolio value is a function of the risk factors $\mathbf{P}$: $V(\mathbf{P})$. The change in the portfolio value $\Delta V(\mathbf{P})$ with respect to changes in each of the risk factors $\mathbf{\Delta P}$ is given by:

$$\Delta V(\mathbf{P}) := V(\mathbf{P} + \mathbf{\Delta P}) - V(\mathbf{P}). \tag{6}$$

We can write the *first order Taylor approximation* as:

$$V(\mathbf{P} + \mathbf{\Delta P}) \cong V(\mathbf{P}) + \sum_{i=1}^{n} \frac{\partial V}{\partial P^{(i)}} \cdot \Delta P^{(i)} \tag{7}$$

The *delta-equivalents* of the portfolio to the risk factors are defined as

$$\delta_i := \partial V / \partial P^{(i)} \cdot P^{(i)}. \tag{8}$$

The partial derivatives are the *sensitivities* of the portfolio with respect to the risk factors. Sensitivities can be computed via *sensitivity analysis*, see Section 4. The change in portfolio value can be expressed as:

$$\Delta V(\mathbf{P}) \cong \sum_{i=1}^{n} \frac{\partial V}{\partial P^{(i)}} \cdot P^{(i)} \cdot \frac{\Delta P^{(i)}}{P^{(i)}} = \sum_{i=1}^{n} \delta_i r^{(i)} = \delta' \mathbf{r} \tag{9}$$

where $r^{(i)} = \Delta P^{(i)} / P^{(i)}$. Since the Equation 5 states that the change in risk factors is a multivariate normal $\mathcal{N}_n(0, \mathbf{\Sigma})$, then the change in the portfolio value is normally distributed with expected value 0 and variance $\delta'\mathbf{\Sigma}\delta$:

$$\Delta V(\mathbf{P}) \sim \mathcal{N}(0, \sqrt{(\delta'\mathbf{\Sigma}\delta)}) \tag{10}$$

Remark. For the *SST* the portfolio value is given by the Risk Bearing Capital (RBC). The RBC is defined as the market value of assets minus the best estimate of liabilities (see [1], Art. 47).

Two risk measures are usually defined on the change in portfolio value ΔV:

Value at Risk (VaR) The Value at Risk of ΔV at confidence level α is defined as the solution of the following equation:

$$\mathbb{P}[\Delta V \leq VaR_\alpha(\Delta V)] = \alpha \tag{11}$$

Conditional VaR ($CVaR$) The $CVaR$ of ΔV at level α is defined as

$$CVaR_\alpha(\Delta V) = \mathbb{E}[\Delta V | \Delta V < VaR_\alpha(\Delta V)] \tag{12}$$

Remark. For the *SST* the level $\alpha = 1\%$ and the $CVaR$ as risk measure are chosen by the Supervisory Authority (FOPI)(see [1], Art. 41).

3 Beyond the linear approximation

Let us go beyond the first order approximation[2]. In this case the change in the portfolio value ΔV can be expressed as:

$$\begin{aligned} \Delta V(\mathbf{P}) &\cong \\ &= \sum_{k=1}^{n} P_k \frac{\partial V}{\partial P_k} \cdot \frac{\Delta P_k}{P_k} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} P_i P_j \frac{\partial^2 V}{\partial P_i \partial P_j} \cdot \frac{\Delta P_i}{P_i} \frac{\Delta P_j}{P_j} \\ &= \sum_{k=1}^{n} P_k \frac{\partial V}{\partial P_k} r_k + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} P_i P_j \frac{\partial^2 V}{\partial P_i \partial P_j} r_i r_j \end{aligned} \tag{13}$$

where $r_k = \frac{\Delta P_k}{P_k}$, $\delta_k = P_k \frac{\partial V}{\partial P_k}$, $\gamma_{ij} = P_i P_j \frac{\partial^2 V}{\partial P_i \partial P_j}$ and n is the number of risk factors. Thus we have:

$$\begin{aligned} \Delta V(\mathbf{P}) &\cong \\ &= \sum_{k=1}^{n} \delta_k r_k + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} r_i r_j \\ &= \delta' \mathbf{r} + \frac{1}{2} \mathbf{r}' \Gamma \mathbf{r} \end{aligned} \tag{14}$$

[2] For similar introductions see: [2], [6] and [7].

In this framework, the triplet $(\delta, \Gamma, \mathbf{\Sigma})$ is called the *portfolio structure*. We refer once again the reader to Section 4 for the computation of the partial derivatives.

We have the following theorem[3]:

Theorem 3.1. *Let us assume the hypothesis of Sections 2 and 4. Given the portfolio structure* $(\delta, \Gamma, \mathbf{\Sigma})$ *such that*

$$\Delta V = \delta' \mathbf{r} + \frac{1}{2} \mathbf{r}' \Gamma \mathbf{r} \tag{15}$$

the first two moments *of* ΔV *are given by:*

$$\begin{aligned} \mathbb{E}(\Delta V) &= \frac{1}{2} tr[\Gamma \Sigma] \\ var(\Delta V) &= \delta' \Sigma \delta + \frac{1}{2} tr[\Gamma \Sigma] \end{aligned} \tag{16}$$

Let X *be the 'normalized'* ΔV*:*

$$X := \frac{\Delta V - \mathbb{E}(\Delta V)}{\sqrt{(var(\Delta V))}} \tag{17}$$

The moments for $i > 2$ *are:*

$$\begin{aligned} \mathbb{E}[X^3] &= \frac{\frac{1}{2} 3! \delta' \Sigma [\Gamma \Sigma] \delta + \frac{1}{2} 2! tr([\Gamma \Sigma]^3)}{var(\Delta V)^{3/2}} \\ \mathbb{E}[X^4] &= \frac{\frac{1}{2} 4! \delta' \Sigma [\Gamma \Sigma]^2 \delta + \frac{1}{2} 3! tr([\Gamma \Sigma]^4)}{var(\Delta V)^2} \end{aligned} \tag{18}$$

or, in general,

$$\mathbb{E}[X^i] = \frac{\frac{1}{2} i! \delta' \Sigma [\Gamma \Sigma]^{i-2} \delta + \frac{1}{2} (i-1)! tr([\Gamma \Sigma]^i)}{var(\Delta V)^{i/2}} \tag{19}$$

This theorem allows us to apply the *Cornish-Fischer approximation* exposed in the Section 3.2.

3.1 Delta-Gamma Monte Carlo

According to our assumptions, the n economic risk factors are not independent. The Equation 5 states that the dependency structure is (fully) described by the correlation matrix $\mathbf{\Sigma}$.

We can then define the Cholesky decomposition of a positive definite matrix $\mathbf{\Sigma}$ as the triangular matrix C that verifies the equation:

$$C'C = \mathbf{\Sigma}. \tag{20}$$

[3] See [6] and [7]

Let $\mathbf{r}_I$ be a multivariate Gaussian $\mathcal{N}_n(0, I)$. Then the random vector $\mathbf{r} := \mathbf{r}_I{}'C$ has variance covariance matrix $\mathbf{\Sigma}$. The proof is straightforward.

Once computed C, it is then possible to:

1. generate n *independent* normally distributed random samples: $\hat{\mathbf{r}}_I$
2. multiply $\hat{\mathbf{r}}_I{}'C = \hat{\mathbf{r}}$.

The vector $\hat{\mathbf{r}}$ is called a *scenario*. Once $\hat{\mathbf{r}}$ has been generated, it is possible to apply the portfolio structure $(\delta, \Gamma, \mathbf{\Sigma})$ to compute the change in portfolio value $\Delta V(\hat{\mathbf{r}})$ corresponding to the scenario $\hat{\mathbf{r}}$. Repeat the simulation m times to obtain an *empirical distribution* of the change in portfolio value ΔV.

The risk measures, VaR_α and $CVaR_\alpha$ at a given confidence level α can be computed from the order statistics of the losses L^k:

$$L^1 \leq L^2 \leq \ldots L^k \leq \ldots \tag{21}$$

From these statistics we can just extract the first α samples. For m simulations we have to select the first $m \cdot \alpha$ order statistics. The maximum of these values is the quantile at α, i.e. the VaR_α, and the average is the $CVaR_\alpha$.

3.2 The Cornish-Fisher Approximation

Theorem 3.1 in the Section 3 shows how to compute the first four moments of the change in portfolio value ΔV, here denoted as:

$$\mu_1 = \mathbb{E}(\Delta V) \qquad \mu_2 = \sigma^2 = var(\Delta V) \tag{22}$$

$$\mu_3 = \frac{\mathbb{E}(\Delta V - \mu_1)^3}{\sigma^3} \qquad \mu_4 = \frac{\mathbb{E}(\Delta V - \mu_1)^4}{\sigma^4} \tag{23}$$

Let z_α be the α-quantile of the standard Gaussian distribution and denote with ρ_3 and ρ_4 the standardized moments of ΔV:

$$\rho_3 = \frac{\mu_3}{\sqrt{\mu_2}^3} \quad \text{and} \quad \rho_4 = \frac{\mu_4}{\mu_2^2} - 3. \tag{24}$$

The Cornish-Fisher approximation of the $\alpha-$quantile of ΔV is given by:

$$\tilde{q}_\alpha(\Delta V) \cong \tilde{z}_\alpha \cdot \mu_2 + \mu_1 \tag{25}$$

where

$$\tilde{z}_\alpha \cong z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)\rho_3 + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)\rho_4 - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)\rho_3^2 \tag{26}$$

Thus, the quantiles of ΔV can be estimated directly, without an explicit approximation of the entire distribution. Hence we have an estimation for the VaR_α for every confidence level α. The relationship between VaR and $CVaR$ is given[4] by:

$$CVaR_\alpha = -\frac{1}{\alpha}\int_\alpha^1 VaR_z dz. \tag{27}$$

Solving the integral numerically, we have a Cornish-Fisher estimation of the $CVaR$:

$$CVaR_\alpha \cong \frac{1}{n}\sum_{i=0}^{n-1} \tilde{q}_{\alpha-i}(\Delta V). \tag{28}$$

The Cornish-Fisher approximation is known to be not very precise, especially in the tail of the distribution. However it is a good way to 'backtest' the results of the Monte Carlo simulation. The $CVaR$ computed by the Monte Carlo simulation and the Cornish-Fischer approximation can be different.

4 Sensitivity analysis

First and second derivatives can be computed using the change in portfolio value with respect to changes in the risk factors. The quantities reported to FOPI are the change in portfolio value for a change in one risk factor:

$$\begin{array}{rl} \text{up} & V(P^{(i)}+\Delta P^{(i)}) - V(P^{(i)}) \\ \text{and down} & V(P^{(i)}-\Delta P^{(i)}) - V(P^{(i)}). \end{array}$$

For the first derivatives we have:

$$\frac{\partial V}{\partial P_i} \cong \frac{[V(P^{(i)}+\Delta P^{(i)}) - V(P^{(i)})] - [V(P^{(i)}-\Delta P^{(i)}) - V(P^{(i)})]}{2\Delta P^{(i)}} \tag{29}$$

The second derivatives are approximated by:

$$\frac{\partial^2 V}{\partial P_i^2} \cong \frac{[V(P^{(i)}+\Delta P^{(i)}) - V(P^{(i)})] + [V(P^{(i)}-\Delta P^{(i)}) - V(P^{(i)})]}{\Delta P^{(i)}\cdot\Delta P^{(i)}} \tag{30}$$

Notice that the Equation 30 gives only the second derivatives with respect to the same variable, i.e., only the diagonal of the matrix Γ.

[4]See [5].

Proof. For the proof consider a differentiable function f of n variables $x, y, z, ...$:

$$f(x, y, z, ...). \tag{31}$$

Without loss of generality we report the proof referring to the first variable only. The *first order* Taylor approximation is:

$$\begin{aligned} f(x+h, y, z, ...) &\cong f(x, y, z, ...) + \frac{\partial f}{\partial x} \cdot h \\ f(x-h, y, z, ...) &\cong f(x, y, z, ...) - \frac{\partial f}{\partial x} \cdot h \end{aligned} \tag{32}$$

The *difference* of these two equations:

$$f(x+h, y, z, ...) - f(x-h, y, z, ...) \cong 2\frac{\partial f}{\partial x} \cdot h \tag{33}$$

Thus we have:

$$\begin{aligned} &\frac{\partial f}{\partial x} \cong \frac{f(x+h, y, z, ...) - f(x-h, y, z, ...)}{2 \cdot h} \\ &= \frac{\Big[f(x+h, y, z, ...) - f(x, y, z, ...)\Big] - \Big[f(x-h, y, z, ...) - f(x, y, z, ...)\Big]}{2 \cdot h} \end{aligned} \tag{34}$$

which is the Equation 29. The *second order* Taylor approximation is:

$$\begin{aligned} f(x+h, y, z, ...) &\cong f(x, y, z, ...) + \frac{\partial f}{\partial x} \cdot h + \frac{1}{2}\frac{\partial^2 f}{\partial x^2} \cdot h^2 \\ f(x-h, y, z, ...) &\cong f(x, y, z, ...) - \frac{\partial f}{\partial x} \cdot h + \frac{1}{2}\frac{\partial^2 f}{\partial x^2} \cdot h^2 \end{aligned} \tag{35}$$

where obviously $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x}(x, y, z, ...)$ and $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x^2}(x, y, z, ...)$. The *sum* of these two equations is:

$$f(x+h, y, z, ...) + f(x-h, y, z, ...) \cong 2f(x, y, z, ...) + \frac{\partial^2 f}{\partial x^2} \cdot h^2 \tag{36}$$

Thus we have:

$$\begin{aligned} &\frac{\partial^2 f}{\partial x^2}(x, y, z, ...) \cong \frac{f(x+h, y, z, ...) - f(x-h, y, z, ...) - 2f(x, y, z, ...)}{h^2} \\ &= \frac{\Big[f(x+h, y, z, ...) - f(x)\Big] + \Big[f(x-h, y, z, ...) - f(x)\Big]}{h^2} \end{aligned} \tag{37}$$

which is the Equation 30.

□

Remark. The sensitivities delivered for the SST do not allow us to compute the cross derivatives

$$\frac{\partial^2 f}{\partial x \partial y} \tag{38}$$

because we would need the *joint* changes in portfolio value with respect to a change in *two* risk factors at the same time:

$$f(x+h, y+k, \dots) - f(x, y, \dots). \tag{39}$$

Obviously the company can compute these quantities simply re-evaluating the portfolio.

Since these data are not available we need the following assumption.

Assumption 4.1 (Diagonal Gamma). *We assume that the matrix Gamma is a diagonal matrix:*

$$\Gamma := \begin{bmatrix} \gamma_{11} & 0 & \dots & \dots & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & \gamma_{ii} & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \dots & \dots & 0 & \gamma_{nn} \end{bmatrix}$$

A comment is in order here. The 'Diagonal Gamma' Assumption 4.1 is verified for bonds where the cross derivatives are null for two different spot risk-free rates:

$$\frac{\partial^2 f}{\partial x \partial y} = 0 \text{ for } x \neq y.$$

The same is for true shares, usually modeled linearly. The second partial derivatives are not null when the portfolio contains financial derivatives:

$$\frac{\partial^2 f}{\partial x \partial y} \neq 0 \text{ for } x \neq y$$

The second partial derivatives are again not null in case of investment in other currencies because of the multiplication with exchange rates. Some information is lost with this additional assumption.

5 Numerical results

The Monte Carlo simulation has been realized generating 50000 scenarios. The companies considered partecipated in the SST Field Test

2006. They are summarized in the Table 1. The companies for which the sensitivities to the predefined FOPI's risk factors are not available areexcluded. Usually these companies have *full internal models*.

	Health	P&C	Life	Total
Number Of Companies F.T.'06	13	19	15	47
Excluded	1	5	3	9
Number Of Companies Considered	12	14	12	38

Table 1: Companies considered for the numerical analysis. The first line is the number of companies taking part to the Field Test 2006.

The output of the Monte Carlo simulation can be represented as in the Figure 1. In order to keep the complete confidentiality of the data, the x axis reports only the value in 0. The distribution in light color is the Gaussian approximation. As expected, it is symmetric and centered in zero. The Delta-Gamma approximation is in dark color. The non-zero mean is clearly visible. The bottom-left frame shows a particular of the tails.

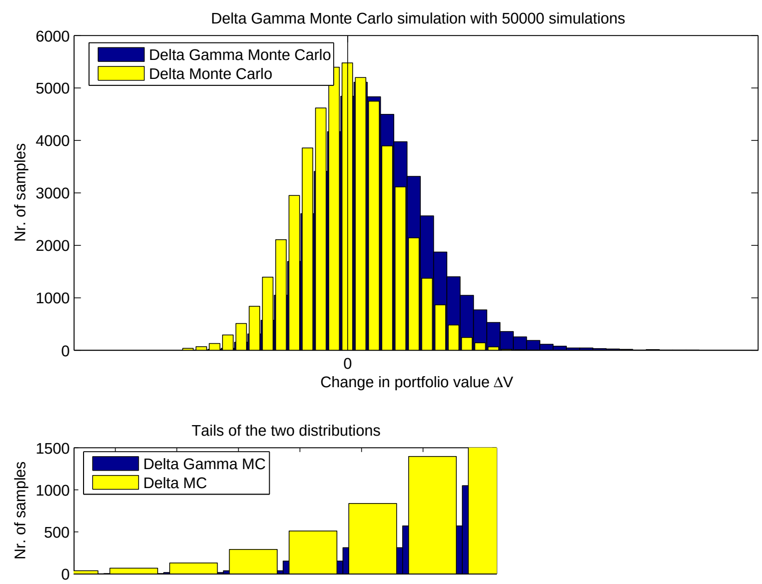

Figure 1: Output of a Monte Carlo simulation for a randomly chosen P&C company

As expected, the Delta-Gamma approach produces more skewed (see Figure 1) and heavy tailed distributions (see Figure 2) for the ΔV. By the point of view of risk (and capital requirement) the left

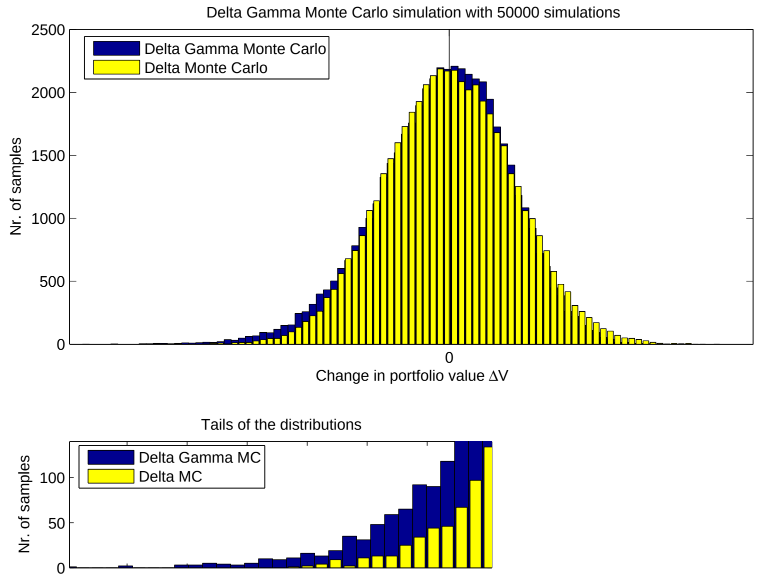

Figure 2: Output of a Monte Carlo simulation for a randomly chosen Life company

tail is the interesting one. In the Figure 3 you can see the tail of four randomly chosen Life companies compared to P&C/Health companies.

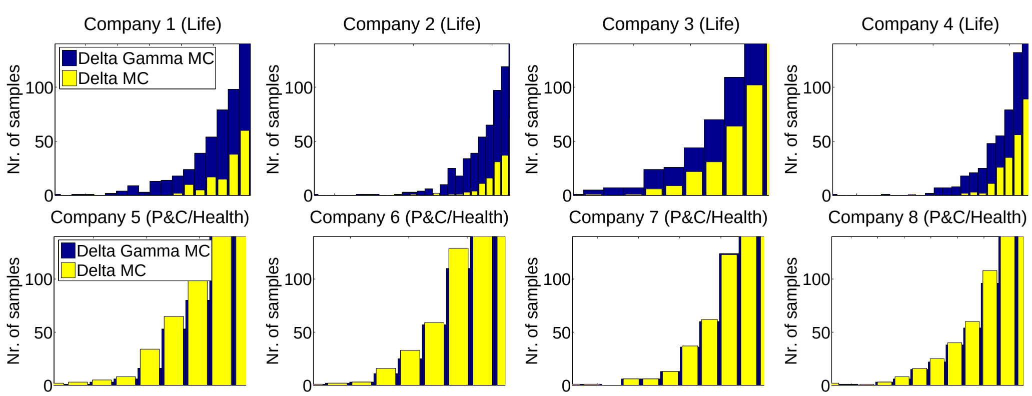

Figure 3: Left tail of the change in portfolio value for four randomly chosen Life Companies (first line) and P&C/Health Companies (second line).

The convexity captured by the Delta-Gamma approach can produce a substantial change in kurtosis. That agrees with the Equation 18. However this effect is higher for Life companies than for the others lines of business.

In order to confirm that, we plot the difference between the $CVaR$

in the Gaussian case (*Delta* approach) and the Delta-Gamma approach in the Figure 4. The $CVaR$ from the Gaussian approximation (Monte Carlo with the Delta approximation only) is the *light bar*. The *dark bar* is the Delta-Gamma approximation via the Cornish-Fisher estimation as in the Section 3.2. We apply the Formula 28 with $n = 100$. The *dark dot* is the 'true' value obtained with the Delta-Gamma Monte Carlo simulation.

The market risk is slightly overestimated (by the point of view of Delta-Gamma) for Health and P&C companies. For Life portfolio the $CVaR$ is underestimated from 0 to more than 40 percent.

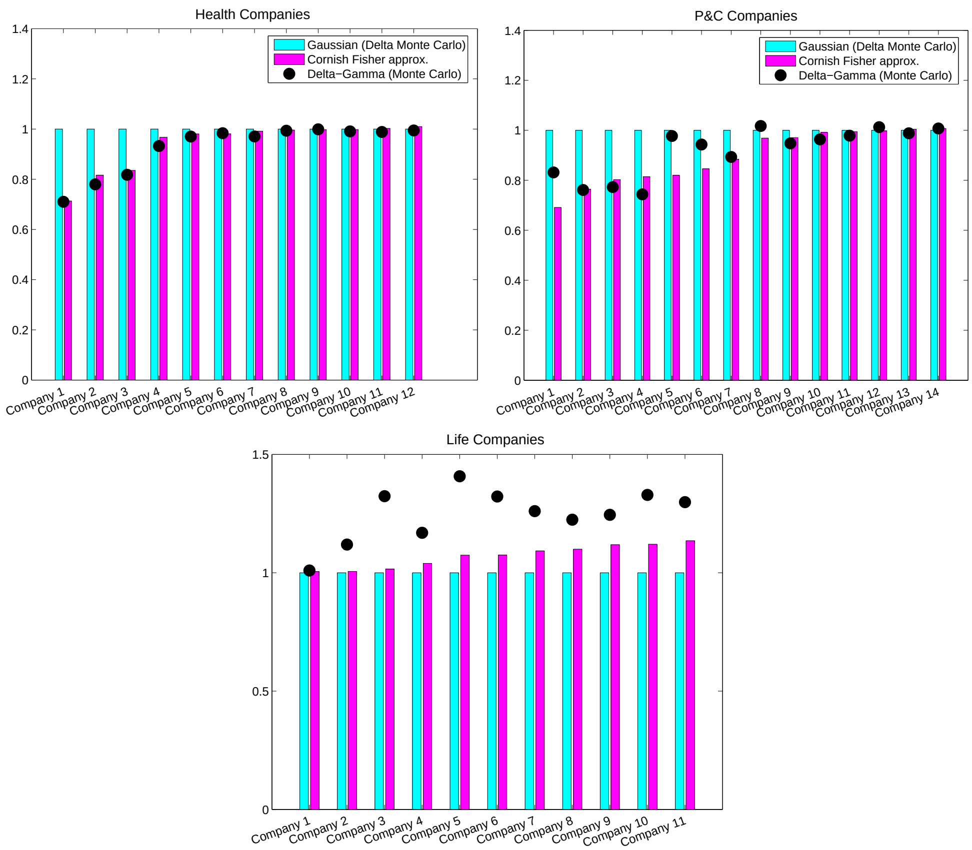

Figure 4: Comparison of the $CVaR_{1\%}$ for the Gaussian and the Delta-Gamma approach. The light line is the Gaussian $CVaR$ normalized to one. The dark line is the approximation of the $CVaR_{1\%}$ using the Formula 27. The bullet is the Delta-Gamma approximation with the Monte Carlo simulation.

The results show that the SST Standard Model is unsuitable for *all* considered Life companies since it substantially *underestimates* risk.

6 Conclusions and further research

This paper can be summarized as follows. The SST Standard Model assumes a $(\delta, \mathbf{\Sigma})$ portfolio structure. A model is robust when a small change in the assumptions does not produce big changes in the results.

So we modify the portfolio structure from $(\delta, \mathbf{\Sigma})$ to $(\delta, \Gamma, \mathbf{\Sigma})$. The results is that the model is usually not robust for Life portfolios.

As a consequence, the linear approximation in a variance-covariance approach is strongly discouraged for such portfolios. The change in portfolio value ΔV is not Gaussian but heavy tailed.

Much more sophisticated methods have to be introduced to capture such a tail behavior. We discourage the use of fast approximations, like the Cornish-Fisher method, even if they are widely used by banks. The reason is that they refer to totally different kinds of portfolios, time horizons and levels of confidence. We suggest to rely on Monte Carlo simulations.

A last warning. The methodology used for this analysis refers mainly to standard portfolios. That is, portfolios with standard Life contracts. When options are embedded in Life insurance contracts, other precautions have to be taken.

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