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Technical document on the Swiss Solvency Test

Federal Office of Private Insurance

Version of 2 October 2006

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1. Preliminary remarks

This document introduces the technical aspects of the Swiss Solvency Test (SST).

This version of the document does not contain the following topics:

- valuation of the assets,
- modelling of the market risks in the standard model.

These topics will be addressed in separate documents.

The structure of this document is:

- In Chapter 2, the principles of the SST are introduced first, and then the terms "risk-bearing capital" and "target capital" are defined in detail.
- Chapter 3 discusses the market-consistent valuation of assets and liabilities.
- Chapter 4 describes the standard model for life, health, non-life, and accident insurers.
- Chapter 5 illustrates the scenarios.
- Chapter 6 discusses the market value margin.
- Chapters 7 and 8 contain references and technical appendices.

Changes since the version of 13 June 2006:

- Correction of the correlation matrix for the risks of the biometric parameters of life insurance: 0.75 instead of 0.5.
- More details concerning extreme events in accident insurance provided in section 4.4.8.2.
- For the description of the market risk model, the reference to the market risk document has been supplemented by a short explanation.
- Corrections in appendix 8.6.
- Cosmetic changes in section 4.5 on health insurance risks..
- Correction in the "List of the predefined scenarios": Accident scenarios must also be evaluated by health insurers.
- Correction in SST Life: standard deviation of the probability of exercise of options harmonized with Template 2006.
- More details provided in section 4.4.11.

2. Principles of the SST

2.1. Introduction

> The goal of the Swiss Solvency Test (SST) is to obtain a picture of 1) the amount of risks borne by an insurance undertaking, and 2) its financial capacity to bear these risks. The amount of the risk assumed is measured with the target capital (TC), and the capacity to bear risks is measured with the risk-bearing capital (RBC).

By comparing the risk-bearing capital and the target capital, insurance undertakings and the supervisory authority gain knowledge on the financial situation of the insurance undertakings.

The SST is based on the following basic principles.

2.1.1. The principles of the SST

All assets and liabilities must be valuated on a market-consistent basis. The difference between the market-consistent value of the liabilities and the discounted best estimates of their associated payment flows is called market value margin (MVM).

The risks to be examined are market, credit, and insurance risks.

The available capital is given by the risk-bearing capital (RBC). It is defined as the difference between the market-consistent values of the assets and the discounted best estimates of the liabilities.

The required capital is given by the target capital (TC). It is defined as the sum of the market value margin and the expected shortfall of the difference between the discounted RBC in one year and the current RBC.

The market value margin is approximated by the cost-of-capital approach. This is the sum of discounted costs of capital for future required regulatory capital for the run-off of the portfolio arising from liabilities and assets replicated to the extent possible.

The risk-bearing capital must be greater than or equal to the target capital.

The SST applies to individual legal entities and to groups and conglomerates with head offices in Switzerland.

The insurance undertakings must evaluate a series of scenarios. These consist of (i) scenarios predetermined by the supervisory authority, and (ii) scenarios specific to the undertaking. If risks described by the scenarios are not taken into account in the risk model, then the results from the evaluation of the scenario must be incorporated into the target capital.

Uncertain values must be treated stochastically.

Risk models developed by the undertakings ("internal models") may and should be used. Such models may partially or entirely replace the standard model. An internal model must be used for risks that are not adequately described by the standard model.

The internal model must be integrated into the risk management processes of the undertaking.

The structure and the assumptions of the internal model must be published. The scope of the publication must be such that an external person with specialized knowledge can form a qualified opinion about the model and its quality.

The insurance undertaking must draft an SST report. This report must permit an external person with specialized knowledge to understand the results of the SST. The report must be signed by the general management.

The general management of an insurance undertaking is responsible that the undertaking complies with the aforementioned principles of the SST.

2.1.2. The status of the standard model

Insurance undertakings required to perform the SST must fulfil the principles mentioned. In addition to these principles, there is an SST standard model for health, life, and non-life insurers. The standard model was developed by the supervisory authority in close collaboration with the insurance industry,

institutes of higher education, and other interested circles. The standard model consists of a model structure and parameters, and it applies to business in Switzerland. It can be used primarily by insurers whose structure of assets and insurance products is not too complicated.
Insurance undertakings whose risks are not sufficiently described by the standard model must expand the model or replace it with their own internal model. In particular, this concerns groups of insurance undertakings, insurers with business activities abroad, and reinsurers.
The standard model can be expanded easily, thanks to its modular structure.

2.1.3. Time frame of the calculations

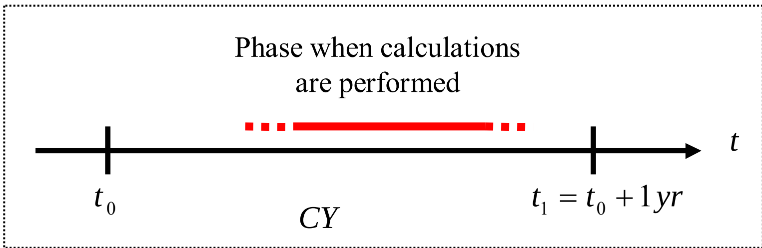
The SST calculates the risk of the portfolio of assets and liabilities existing at time t_0. As a rule, t_0 is the beginning of the first January of the year in which the SST is performed. This year is called "current year" (CY). An exception applies if the risk situation or the available capital of an insurance undertaking changes dramatically over the course of a year. In this event, a new SST must be performed.

In many cases, the asset and liabilities portfolio on 1 January is similar to the portfolio on 31 December of the preceding year. For this reason, the end-year items may be used to evaluate the risks and the available capital. This has the advantage that the items have been attested by auditing companies and do not change over time.

If, however, significant differences exist between the risk situation on 1 January and the preceding 31 December, for instance due to acquisition of a client base, then the new risk situation on 1 January is decisive. The portfolio on 31 December may still be used, but it must be adjusted by the change.

The risk assessed is how strongly the value of a portfolio can change over the course of one year. The end of this time period is generally the end of 31 December and is designated $t_1 = t_0 + 1yr$.

The insurer cannot be expected to perform the SST on 1 January of the year in question. Instead, the calculations are performed over the course of the year.



2.2. Risk-bearing capital

The risk-bearing capital (RBC) is the capital that can be used to equalize fluctuations over the course of business. The values taken into account for the risk-bearing capital may not be used for other purposes. The RBC is defined as the difference between the market-consistent value of the assets and the discounted best estimate of the liabilities

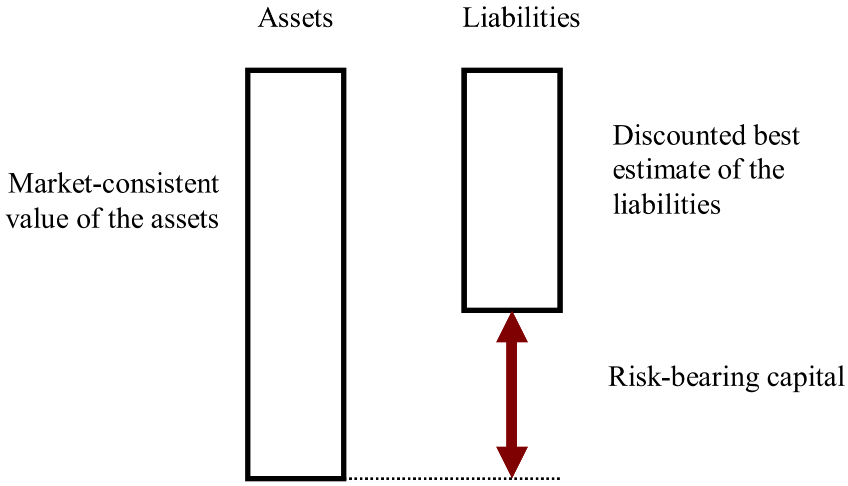


Figure: Definition of risk-bearing capital (RBC) as the difference of values between assets and liabilities at time t.

Chapter 3 will provide a more exact definition of market-consistent assets and the discounted best estimate of the liabilities.

The demands on the minimum risk-bearing capital at time t_0 are represented by the amount of the target capital (ZK) (TC).

2.3. Risk

2.3.1. Types of risk examined

The risks to be measured are technical risks, market risks, and credit risks. Operational risks are currently not considered by the SST with respect to capital requirements. They may be included in the future, however.

> The market risk is the risk that the RBC may change due to changes of external economic factors or influences. These influences are called risk factors. In the standard model of the SST, nearly 100 risk factors in the areas of interest rates, shares, real estate, and alternative investments are examined.
>
> The technical risk is the risk that the RBC may change due to the randomness of the insured risks and the uncertainties in estimating technical parameters.
>
> The credit risk is the risk that the RBC may change due to defaults and rating changes of the counterparties. In particular, credit risk is contained in bonds, loans, guarantees, mortgages, and reinsurance policies and balances.

2.3.2. Time horizon: 1 year

The risks examined arise from items that generally exist over very different time periods. While some asset items can be converted into cash within days, there are other assets and liabilities to which the insurer is bound for years or decades. The insurance industry therefore often chooses 1 year as a characteristic time period over which risks are measured. The SST adopts this convention.

2.3.3. Risk-bearing capital at the end of the year, definition of risk

Figure 1 represents the risk-bearing capital at the beginning (t_0) and at the end (t_1) of the year. The risk-bearing capital at time t_0 can be derived from the enumeration of assets and liabilities, i.e., from the market-consistent balance sheet, and is therefore known ($RTK(0)$) (RBC (0)). The future risk-bearing capital ($RTK(1)$) (RBC (1)), however, is an unknown, i.e. stochastic, quantity, since the environment in which the undertaking is situated will change in an unknown way.

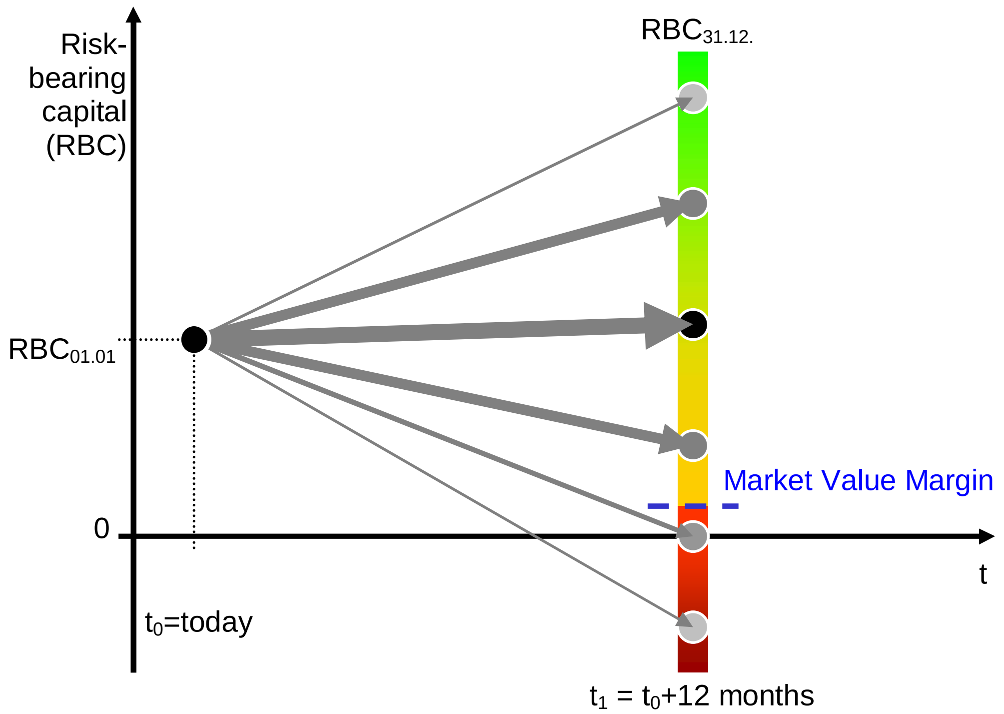


Figure 1: Risk-bearing capital at times t_0 (known quantity) and t_1 (unknown, stochastic quantity).

Depending on the magnitude of the RBC at the end of the year, the relationship between the market value of the assets and the value of liabilities will be different:

RBC < 0	Assets < Best estimate of liabilities
0 < RBC < MVM	Best estimate of liabilities < Assets < Market value of liabilities
MVM < RBC	Market value of liabilities < Assets

If the RBC at the end of the year is greater than the market value margin, then the value of the assets is greater than the market value of the liabilities.

Figure 2 examines the different areas of the RBC in more detail.

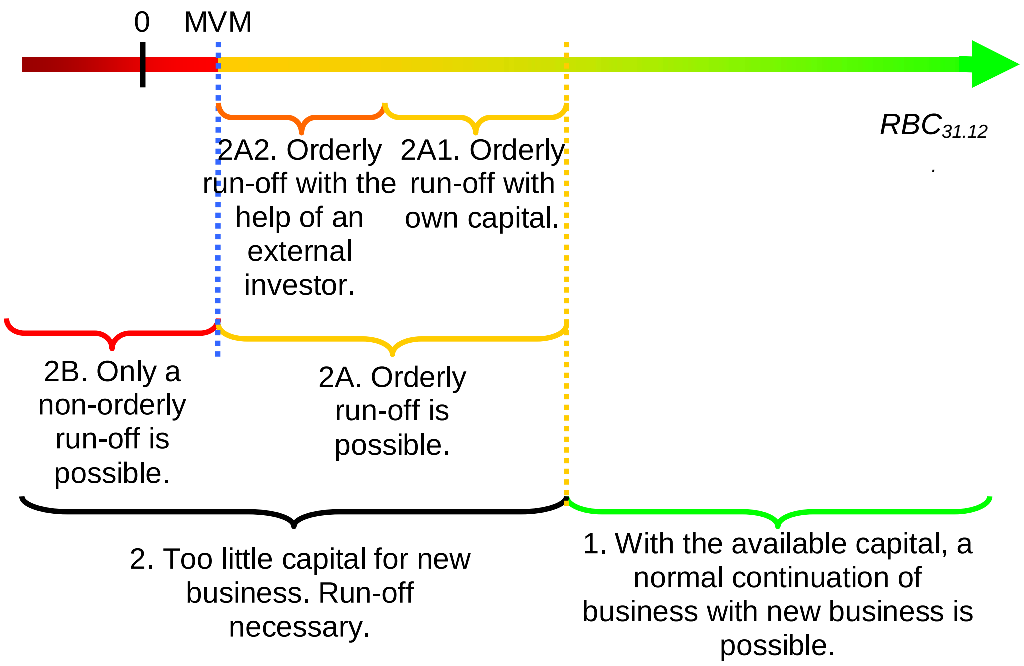


Figure 2: Different magnitudes of the RBC and their effect on the further course of the insurance undertaking.

- Area 1: If the RBC exceeds a certain amount, sufficient RBC is available to bear existing risks and to underwrite new business.
- Area 2: If the RBC does not reach the amount mentioned for Area 1, too little capital is available to take on new business. This means that existing contracts and claims are settled. Depending on whether the RBC is greater or smaller than the market value margin, the run-off risk must be borne by the insurance undertaking, or it is borne by the still existing capital or even by an external capital provider:
    1. Area 2A: The portfolio is in run-off, but the policyholders will most probably receive their guaranteed benefits. In the case of 2A1, the still existing RBC bears the run-off risk. In the case of 2A2, it is possible to transfer the risk to an external capital provider. The reason is that the RBC is greater than the market value margin, i.e., the market value of the assets is greater than the market value of the liabilities. This means that an investor or another insurance is willing to assume the assets and liabilities.
    2. Area 2B: The portfolio being settled does not have sufficient capital (RBC<MVM) for the settlement risks to be borne by the RBC or for an external capital provider to assume the risk. Accordingly, the processing risk remains with the policyholders.
    If the RBC is positive, the expected value of the liabilities is smaller than the value of the assets, but the risk that the liability payments could exceed this value is high. If the RBC is negative, then not even the expected value of the liabilities is covered by the assets.

2.4. Target capital: Measuring the risk

Area 2B mentioned above contains the circumstances in which a very high probability exists that the insurance company will not or cannot meet its obligations relating to existing policyholders. If the policyholder is to be protected, these circumstances must be avoided.

The capital requirements (the target capital) of the SST is therefore chosen so that a situation falling within Area 2B is highly unlikely to arise.

The following section introduces the expected shortfall. The expected shortfall serves to capture the possible low values of the RBC at the end of the year in a single value. This value is the average of the lowest possible RBCs and can therefore be regarded as a representative of these low values. The demands on the current RBC are fixed so that the expected shortfall is no lower than the market value margin.

2.4.1. Expected shortfall

Before we look at the definition of target capital, we will introduce the two risk measures "value at risk" (VaR) and "expected shortfall" (ES). The term "expected shortfall" is synonymous with "tail value at risk" (TailVaR).

The goal of risk measurement in general is to use an appropriate risk measure to assign a real number to an uncertainty or a quantity with an unknown value, so that the risk exposure of this quantity can be represented. The risk measure used in the SST is the expected shortfall or the TailVaR.

For purposes of introducing the expected shortfall, we will first examine a general random variable X, where the negative values of X are the "bad" values (values that we associate with loss, damage, risk, etc.). We associate positive values of X with profits and returns.
As a first step for defining expected shortfall, the value at risk $VaR_\alpha(X)$ of X with a certainty level of $1-\alpha$ (e.g. 99%) is introduced. VaR is defined as

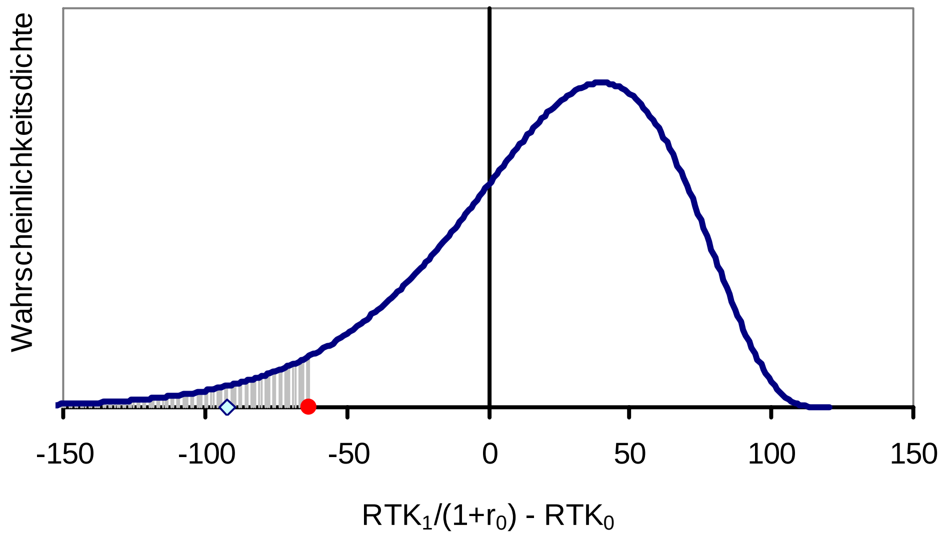
$$VaR_\alpha(X) := \sup(x : P(X \le x) \le \alpha).$$

VaR is the greatest (more precisely the supremum) of all values x for which the probability that X is less than or equal to x is at most equal to α.
(Note: If the distribution function is continuous, then $VaR_{1\%}(X)$ is equal to the value x for which X is less than x in 1% of all cases and greater than x in 99% of all cases.)

The second step now consists in the definition of the expected shortfall (ES) at the certainty level $1-\alpha$ of the random variable X. It is defined as the conditional expected value of X, given that X is less than or equal to the value at risk at the certainty level $1-\alpha$:

$$ES_\alpha(X) = E\left[X \middle| X \le VaR_\alpha(X)\right]. \qquad (1)$$

Occasionally, an event with a probability of occurrence of 1% in one year is called a century event (e.g. flood, storm of the century). This expression is permissible if the risk characteristic does not change over the course of 100 years. For some risks, this is the case (e.g. the number of meteorite impacts), while it is not the case for others (e.g. neck vertebrae injuries in road traffic or avalanche damage related to increasing construction development).



Probability density **$RBC_1/(1+r_0) – RBC_0$**

Figure 3: Representation of the value at risk (VaR, red circle) and the expected shortfall (ES, blue diamond), using the example of a systematic distribution of change to RBC. For the purpose of the figure, the quantile level has been fixed at 5%, not 1%.

The risk measure "expected shortfall" is more conservative than VaR at the same certainty level. Since it can be assumed that a real claims distribution will show several extremely high losses with very low probabilities, the expected shortfall is a more appropriate risk measure, since – in contrast to the VaR – it takes the magnitude of these extreme losses into account.

In contrast to the value at risk, expected shortfall quantifies what the average cost of one of the (100·α)% worst events is. In practice, expected shortfall turns out to be more stable than value at risk. Expected shortfall also exhibits other useful (mathematical) properties of continuous random variables, such as coherence.
(Cf.:

- Artzner, P., Delbaen, F., Eber, J.-M., Heath, D., 1999. Coherent measures of risk. Math. Fin. 9, 3, 203-228 and
- Acerbi, C., Tasche, D. 2000, On the coherence of Expected Shortfall, Journal of Banking and Finance 26(7), 1487-1503).

Occasionally, other authors (e.g. Swiss Re) do not define VaR and expected shortfall as the aforementioned values, but rather as the distance of these values from the expected value of a distribution. Which definition is used is primarily a question of convention and the demands on a risk measure. In particular, the definition depends on how the translation invariance is formulated. In the SST, the definition given above is used throughout. We prefer this formulation to the distance to the expected value, since it provides information on what value should be assigned to the extraordinary circumstances, and not only by what value the extraordinary circumstances deviate from the expected value.

2.4.2. Target capital

It was mentioned above that the circumstances 2B from section 2.3.3 are undesirable. They should be avoided where possible. The target capital is the answer to the question of how large the risk-bearing capital at time t_0 must be for RBC at time t_1 to be greater than or equal to the market value margin with a high degree of probability. Using the expected shortfall, the answer is

$$ES_{\alpha}[RTK(t_1)|RTK(t_0) = ZK] = MVM \tag{2a}$$

$ES_{\alpha}[RBC(t_1)|RBC(t_0)=TC]=MVM$

This is an implicit equation for the target capital TC. It states that if the current $RTK(t_0)$ $RBC(t_0)$ is sufficiently large for purposes of the SST (i.e. equal to the target capital), then the expected shortfall of the RBC is guaranteed to be equal to the market value margin at the end of the year. Accordingly, due to the construction of the expected shortfall, the probability is low that $RTK(t_1)$ $RBC(t_1)$ would fall below the market value margin.

The following simpler but essentially equivalent definition of target capital is used instead of the equation above:

$$ZK = -ES_{\alpha}\left(\frac{RTK(t_1)}{1+r_1^{(0)}} - RTK(t_0)\right) + \frac{MVM}{1+r_1^{(0)}}, \tag{2b}$$

$TC = - ES_{\alpha}[(RBC(t_1)/\ldots - RBC(t_0)] + \ldots$

$r_1^{(0)}$ stands for the current one-year risk-free interest rate.
The target capital is therefore composed of the expected shortfall of the change of the risk-bearing capital for the one-year risk and the market value margin (calculation see section 6).

To cover all the receivables at the end of the year, the RBC at the end of the year is required to be greater than or equal to the market value margin in the average of the α worst cases. This market value margin is set as the price for the risk capital to be held in the future that would have to be paid to another insurance undertaking or investor if they should assume the portfolio. Accordingly, the market value margin essentially covers the costs that a company assuming the portfolio would have to pay to provide the future target capital and can therefore also be considered a risk premium for the run-off of the liabilities.

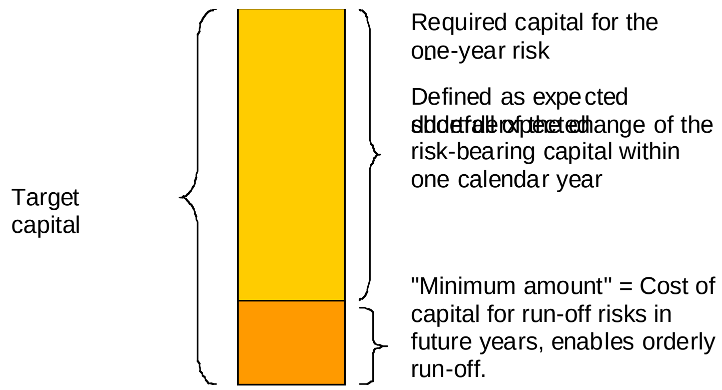


Figure 3: Target capital as the sum of the required 1-year risk capital and minimum capital

In other words, the TC at a certainty level of 99% is the expected value of the 1% largest possible value reductions plus the abovementioned market value margin for future risk capital. If one of the (unlikely) 1% largest value reductions in the RBC then occurs within one year, then on average there is still sufficient RBC to take over the future risk capital. Numerical examples for the amount of the one-year risk and the market value margin are given in Figure 4.

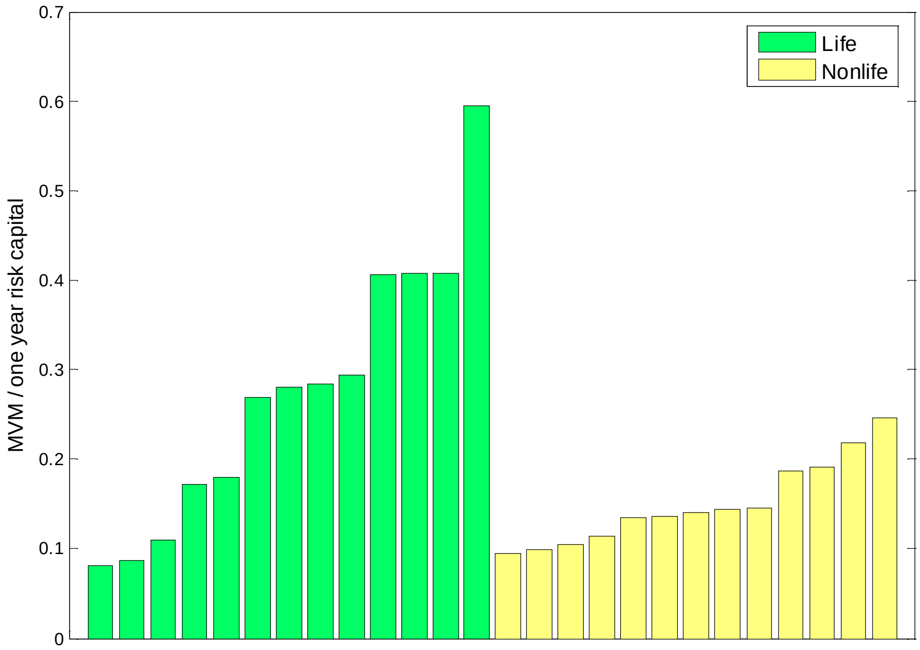


Figure 4: Relationship of the market value margin to the one-year risk capital for individual life and non-life insurers. The data is from the 2005 SST test run.

2.5. Risks in insurance groups and conglomerates

For now, please consult the (publicly accessible) discussion papers on the risks in insurance groups[A].

[A] http://www.bpv.admin.ch/themen/00506

3. Valuation

3.1. Valuation of assets

See the document entitled "Determination of the market-consistent balance sheet values for calculating the risk-bearing capital in the SST".

3.2. Valuation of liabilities for life insurers

The value of the technical liabilities is defined as the expected value (under risk-neutral probability measures) of the future contractually agreed payment flows discounted with the risk-free interest-rate curve. In particular, the best estimate principle must be observed in this regard: The valuation does not contain any implicit or explicit safety, fluctuation, or other loading, but rather refers solely to the expected value of the liabilities.

The risk-free interest-rate curves for Swiss business are defined by Switzerland; equivalent risk-free interest-rate curves for EUR, USD, and GBP business are made available by the supervisory authority.

3.2.1. General notes on modelling liabilities for life insurance

The following cash flows are to be modelled, which must then be discounted using the risk-free interest-rate curve:

Cash inflows:
- Premiums
- Other revenue

Cash outflows:
+ Benefits in the case of death
+ Benefits in the case of survival
+ Annuity benefits
+ Surrender benefits
+ Other benefits (cash)
+ Commissions
+ Administrative costs (including costs for managing capital investments)

In determining these cash flows, the following points must be observed:

- **Biometric and financial risks.** It is assumed that the financial risks are independent of the mortality risk. The independence also approximately holds between financial and disability risks. This is not true with respect to the cancellation rate, which correlates with the interest-rate curve.
- **Interest-rate curve.** The risk-free interest rates are calculated by FOPI and made available to the companies.
- **2nd order foundations.** For biometric risks such as mortality, disability, and reactivation frequencies, the 2nd order foundations must be used ("best estimate" assumptions)
- **Client base.** Only the current client base at valuation time t0 is considered. Future new business is not included. Special assumptions apply to business subject to the Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund (BVG, see below).
- **Segmentation.** The market-consistent valuation of liabilities should, where possible, be conducted at the level of the policy/insured person. However, plausible portfolio compression may also be undertaken.
- **Periodicity.** The points in time should correspond to beginning-of-year data. Sub-annual approaches (semi-annual, quarterly) are also permissible.

- **Horizon.** The projection should extend from the valuation time t0 to the maximum end date of all policies.
- **Reinsurance.** The cash flows must be considered taking reinsurance payments into account.
- **Sub-annual payment flows.** Sub-annual payment flows (triggered by policy surrender, for instance, or the occurrence of the insured event) should be discounted as of the next greater valuation time (beginning of the year).
- **Surpluses.** Surpluses should only be included if they can no longer be reversed (e.g. guaranteed surpluses).
- **Taxes, dividends.** Taxes and dividends should not be taken into account. Only cash flows should be included that "certainly" will occur after the fixed time horizon of one year.
- **Coverage capital, investment returns.** The coverage capital and non-realized investment profits or losses are not included with the cash flows, since no cash actually flows.
- **Costs.** Costs must be projected according to the "going concern" principle. It must be ensured that all costs (including overhead costs) are included.
- **Foreign currency.** Cash flows arising from foreign currency policies must be discounted using the applicable risk-free interest-rate curve.
- **Other revenue.** Other revenue includes returns of commissions for unit-linked life insurance policies.

Other notes are contained in the document "Market-consistent valuation of life insurance liabilities" dated 15 March 2004.

3.2.2. Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund: Notes on modelling occupational pensions

The following text on the Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund (BVG) is a further development of the document entitled "Swiss Solvency Test, Life Insurance, Notes on modelling occupational pensions, Version 0j" and replaces this document.

3.2.2.1. Insurance business to be modelled

In the area of occupational pensions, the parts of the life insurance business must be modelled that are included in the new separate statement of accounts for occupational pensions (in accordance with article 139 of the Supervision Ordinance). If business policy principles give rise to additional risks, then the corresponding guarantees must also be included in the valuation (e.g. intention to offset deficient coverage in a collective foundation or an internal pension scheme).

3.2.2.2. Types of modelling

The expected cash flows of the development of the client base are to be modelled. The modelling should take the contract options into account. With respect to modelling of contract options, please consult the document entitled "Guideline for market-consistent valuation and modelling of options and guarantees for purposes of the Swiss Solvency Test"[B].
The Federal Office of Private Insurance (FOPI) does not prescribe any specific models, with the exception of modelling the interest-rate sensitivity of mandatory retirement assets (see section 8). For modelling other liabilities, both "deterministic" models, in which the development of the client base is realized explicitly, as well as "stochastic" models, which for instance are based on Monte Carlo simulations of the development of the client base, are possible. FOPI does not make any explicit demands on the level of detail of the models. Plausible simplifications are possible and desirable, as long as it is apparent that the risk sensitivity does not change substantially.

[B] This document was developed in a working group of the Swiss Association of Actuaries with the participation of the Federal Office of Private Insurance. It is currently (June 2006) being circulated for consultations and is available at http://www.actuaries.ch/de/forum/documents/Richtlinie-marktnahe-Bewertung-Garantien-OptionenV072_11Apr06.pdf.

3.2.2.3. Development of the client base

The basic assumptions concerning the development of the client base must be presented and justified. The cash flows of a realistic development of the client base according to the current business policy must be modelled. All partial processes of the model must be modelled coherently with the assumed development. The risks and costs as well as the duration of their consideration must likewise fit with the development of the client base. This also applies to the losses pertaining to the annuity conversion rate that arise in accordance with the development of the portfolio.
The development of the client base may also be made dependent on economic conditions.
If the client base is continued normally, then a separate scenario with a heavily decreasing portfolio (e.g. reduction within three years) would be desirable.

3.2.2.4. Separation of mandatory and above-mandatory components

The mandatory and above-mandatory components of the obligations to pay interest on the retirement assets and to convert annuities must be modelled separately.

3.2.2.5. Limitation of the duration of risks in the active lives portfolio

The Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund permits a limitation of the overall duration of consideration of the risks of the active lives portfolio to 10 years, even if the actual development of the portfolio continues beyond this time. This limitation takes into account the possibility for the insurer to improve its ALM in the medium term or to withdraw from the business, as well as the increasing fuzziness concerning the projection of future risks.

3.2.2.6. Minimum interest rate under the Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund

As long as no technical rule for determining the minimum rate under the Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund (BPV) is fixed at the political level, the supervisory authority determines this rule and announces the resulting interest rates.

FOPI rule on the BVG minimum interest rate in the SST:
The BVG minimum interest rate is 70% of the spot interest rate for Confederation bonds with a 7-year term as a rolling average over the last 7 years (abbreviated as 70/7/7). Deviating from this rule, the actual minimum interest rate must be maintained in the first year and the average between the minimum interest rate and 70/7/7 in the second year.

For the SST field test in 2006, FOPI specifies the following values:

(0) Year	(1) Average over 1 year of the spot interest rates Confederation bonds with 7-year term (in %)	(2) 70% of (1)	(3) 70 / 7 / 7	(4) BVG minimum interest rate in the SST (in %)	(5) 120 / 7 / 7
1999	2.630	1.841			
2000	3.710	2.597			
2001	3.162	2.213			
2002	2.877	2.014			
2003	2.159	1.511			
2004	2.323	1.626			
2005	1.853	1.297	1.871		
2006	1.988	1.392	1.807	2.500	
2007	2.036	1.425	1.640	2.070	2.811
2008	2.076	1.453	1.531	1.531	2.625
2009	2.128	1.490	1.456	1.456	
2010	2.194	1.536	1.460	1.460	
2011	2.269	1.588	1.454	1.454	
2012	2.348	1.643	1.504	1.504	
2013	2.427	1.699	1.548	1.548	
2014	2.505	1.753	1.595	1.595	
2015	2.578	1.805	1.645	1.645	

The interest rates for 1999 to 2005 are formed as averages of the daily interest rates. (The figures in column (1) therefore deviate from the spot interest rates as of 1 January as indicated in column (1) of the table labelled "70-7-7 interest rates for the replication portfolio in SST_16-03-06.xls" that was distributed in March.) Forward rates have been chosen as future interest rates, derived from the interest-rate curve in the SST Template.

3.2.2.7. Scenario for the BVG minimum interest rate

For pension plans, a scenario for the BVG minimum interest rate must be modelled that represents a sudden deviation from the interest rate rule. The deviation only applies to the second and third year and fixes the value at 120/7/7:

2007: 2.811%
2008: 2.625%.

A singular effect is modelled without any additional influences. The deviation is meant to take place in the second year with an effect over 2 years, since the first value is determined by the actual minimum interest rate and the Federal Council fixes the minimum interest rate for 2 years in a row. In subsequent years, the 70/7/7 rule is applied again.
When using the replication portfolio for modelling the mandatory retirement assets (see following section), the interest rates of all tranches used must be fixed at 120% (instead of 70%) in the second and third year.

3.2.2.8. Modelling the mandatory retirement assets

The retirement assets must be modelled separately for the mandatory and the above-mandatory portions. All insurers must use the FOPI *Replication Portfolio* described below for modelling the *mandatory* retirement assets, which is implemented in the SST Template. In addition, however, other modellings may also be conducted. If a *cash flow modelling* of the retirement assets is performed that meets the requirements enumerated below, then FOPI will recognize it as equivalent. An SST in which

the mandatory retirement assets is modelled with the replication portfolio must, however, also be performed and submitted in any case.
If the client base changes substantially, then another model must be used in addition to the replication portfolio.

A separate section on annuity conversion follows below.

Characteristics of the replication portfolio:

- It consists of 7 virtual tranches of 7-year Confederation bonds that were issued by the end of 2005 for a time period of 7 years.
- Except for the (mandatory and above-mandatory) retirement assets as of the end of 2005, the replication model has no additional degrees of freedom. The calculation is therefore the same for all life insurers offering occupational pensions.
- Each tranche nominally covers the same share of 1/7 of the retirement assets as of the end of 2005.
- The minimum interest rate is generated with 70% of the coupon returns.

Characteristics of the cash flow model:

- The assets are viewed as a portfolio of reversional life annuities.
- Appropriate portfolio compressions may be undertaken for purposes of simplification.
- The interest paid on the retirement assets corresponds to the above-referenced accrual of the BVG minimum interest rate.
- Either maintenance or aging of the age structure of the portfolio may be assumed, but not a decrease in age.
- The overall portfolio may be maintained or reduced, but not expanded.
- The retirement assets become due upon retirement. If the policy is cancelled, an interest-rate risk reduction may be performed within the first 5 years of the policy.
- The active lives portfolio remaining after 10 years is transferred at the nominal value of the retirement assets.

When modelling the interest obligations of the mandatory retirement assets relating to autonomous collective schemes for which the BVG minimum interest rate need not be complied with, a deviating interest rate that is realistic in terms of business policy may be used. The use must be explained and justified.

3.2.2.9. Modelling the above-mandatory retirement assets

Above-mandatory savings should be modelled according to the business policy. If following the BVG minimum interest rate is not the goal of the business policy, then a deviating interest rate that is realistic in terms of business policy may be used for above-mandatory savings. The use must be explained and justified. The above-mandatory savings may be modelled with the corresponding replication portfolio in the SST Template or with other methods. A limited margin between yield and interest may also be used for the calculation, *but only for a maximum of 10 years.* Also in this case, the method must be explained and justified.

3.2.2.10. Annuity conversion and capital option

The annuity conversion must be modelled separately for the mandatory and the above-mandatory portions.
The retirement capital available for annuities is determined as follows: The retirement assets must be maintained over 10 years according to the development of the client base. Interest in the mandatory portion is paid according to the BVG minimum interest rate (also for liabilities relating to autonomous collective schemes). In the above-mandatory portion, interest is paid in accordance with the interest assumptions for the above-mandatory portion in section 9. Depending on the development of the client base, part of the retirement assets will become available for annuities or for capital withdrawal.
The quota used for capital withdrawal must be explained. The dependence on the interest rate development must be taken into account.

New annuities payments only need to be conducted for 10 years.
For the mandatory portion of the retirement assets that is converted into a retirement annuity, the legal conversion rate must be applied. In accordance with the first revision of the Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund, the BVG annuity conversion rate will fall from 7.2% to 6.8% in 2014. Subsequently, the conversion rate of 6.8% will continue to be applied. With respect to the above-mandatory portion, a 5-year linear transition from the currently approved conversion rate to a second-order conversion rate may be performed.
The retirement annuities can be settled according to the schema in the SST Template (L_BV_Annuities spreadsheet). (In the first version, inadvertently only 9 years were entered. This has been corrected to 10 years.)
The capital option should be valuated according to the "Guideline for market-consistent valuation and modelling of options and guarantees for purposes of the Swiss Solvency Test".

3.2.2.11. Current annuities

The current annuities accrue according to the risk structure and the development of the client base. Their accrual may be limited to a maximum of 10 years, however. The annuities, including those accruing in the future, are discounted to today in a market-consistent manner (2nd order mortality tables, discounted using the interest-rate curve). Accordingly, they are subject to an interest rate risk and a biometric risk in the SST. In the case of current disability annuities, a flat rate can be used to include reactivation in the calculation. The mortality trend is relevant to valuation with an actuarially recognized methodology (generation tables, modelled with the help of the Nolfi approach $q_{x,t} = q_{x,t_0} \cdot \exp(-\lambda_x \cdot (t - t_0))$ and a parameter λ_x that has been determined using a recognized trend estimate procedure).

3.2.2.12. Risk process for active lives

The risk process for active lives can be modelled in a simplified manner with a margin between premiums and claims. We assert that such a margin is possible because of one-year rating, although an adjustment delay and legal restrictions on rate adjustments exist in reality. The margin used should be based on the actual current margin and may take into account certain future possibilities of improvement. However, it may amount to at most 20% of the risk premium and may be used for *at most 10 years*. This margin must of course be subjected to the minimum quota along with the results from the other processes.
The claims correspond to the risk structure of the client base. The premiums can then be derived from the margin arising from the claims.
Naturally, other, finer models are possible.

3.2.2.13. Cost process

The cost process can also be modelled at a flat rate with a margin. The margin used must, however, be based on the current, actual margin. It may improve, but it may also deteriorate; it may amount to at most 20% of the cost premium, and it may be used for *at most 10 years*. This margin must also, of course, be subjected to the minimum quota along with the results from the other processes. If the client base is reduced, increasing cost rates must be taken into account. In any case, the cost development must be explained and justified.

3.2.2.14. Policy cancellation

Policy cancellations and the resulting interest rate losses and losses arising from the elimination of future margins must be realized in accordance with the development of the client base. In addition, the practice in connection with article 53e of the Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund (transfer or retention of current annuities) must be taken into account, i.e. the surrender option must be valuated and subtracted from the risk-bearing capital.
The interest rate sensitivity of the cancellation behaviour must be defined and taken into account in accordance with the "Guideline for market-consistent valuation and modelling of options and guarantees for purposes of the Swiss Solvency Test".

Only policy cancellations within 10 years must be taken into account.

3.2.2.15. Indexing and Cost-of-Living Fund

In the standard model, inflation risks are not taken into account, since an annual adjustment of the inflation premiums is possible. In any case, the resources from the Cost-of-Living Fund may only be used to equalize inflation or transferred to the surplus fund. The Cost-of-Living Fund is processed proportionally to the client base. The current interest rate margin between the agreed interest payments and the actual capital returns may be carried forward at a flat rate. FOPI limits the margin to a maximum of 1%. The margin may be calculated for *at most 10 years*. With respect to settlement, we apply the simplified assumption that the inflation premiums correspond to the amount of the costs and claims.

3.2.2.16. Minimum quota

The effect of the legal rules on the minimum quota should be considered to the extent possible. It should be taken into account that the determination of the minimum quota is based on statutory quantities. The statutory quantities in the account statement are estimated, so that the effect of the minimum quota can be included.
Of course, only those policies are affected that are subject to the minimum quota.

3.2.2.17. Presentation

When presenting the model and the results, the model assumptions, the initial client base, parameters, and important data relating to the client base must be presented as time series. This includes data such as the mandatory and above-mandatory retirement assets, interest, the coverage capital of the current annuities, conversation rate losses, risk premiums, and costs.

3.3. Valuation of liabilities for non-life insurers

The value of provisions and liabilities that are not risk-bearing is composed of

- the best estimate of the cash values of the expected values of the future payments for claims whose claims date is in the past. This includes provisions for IBNyR claims;
- provisions for future costs connected to the claims events mentioned in the first point (ULAE provisions);
- the unearned premium reserve (upr);
- the discounted best estimate value of the additional provisions and liabilities that are not risk-bearing:
    1. bonds issued,
    2. dividend distributions already planned in the previous year,
    3. provisions for any contractual surplus profit participation,
    4. own shares (these are listed on both sides of the balance sheet)
    5. tax provisions
    6. provisions for pensions
    7. other provisions and liabilities that are not risk-bearing.

> The discounted best estimate of the claims provisions is the estimate of the sum of the current cash values of the expected values of the future payments for claims whose claims date is prior to the time of observation. The estimate must be true to expectations and include all information available by the time of observation.

For the determination of the discounted best estimate reserves, it is necessary for each line of business to calculate the best estimates of the future payments and to discount these as of the time of observation (e.g. t_0). The risk-free discount rates $v_j^{(0)}$ must be used. The future payments are calculated via the payment pattern of the undiscounted best estimate reserves. The discounted best estimate reserve at time t_0 is therefore:

$$\sum_{k\geq 1} v_k^{(0)} \beta_{k-1} \cdot R_{PY}^{(0)} = \sum_{k\geq 1} d_{PY}^{(0)} \cdot R_{PY}^{(0)} \ , \tag{3}$$

where $R_{PY}^{(0)}$ are the undiscounted required claims provisions at time t_0 for the observed claims (claims dates in PY). The Swiss Association of Actuaries (SAA) provides guidelines in this regard. The coefficients $(\beta_k)_{k\geq 0}$ designate the payment pattern for each line of business and can be determined depending on the enterprise. Alternatively, the SST suggests standard payment patterns for most lines of business. In this case, the payment patterns $(\alpha_k)_{k\geq 0}$ are independent of the year of occurrence and are derived from the patterns of the large portfolios in the Swiss insurance market. To calculate $(\beta_k)_{k\geq 0}$ from these payment patterns, they must first be rescaled and applied to the reserves at the end of the preceding year CY-1, by year of occurrence, according to the already processed years. The $(\beta_k)_{k\geq 0}$ can then be derived from the development of the total reserve for all years of occurrence. The standard values for the payment patterns $(\alpha_k)_{k\geq 0}$ can be drawn from the SST Template.

3.3.1. Special case of accident insurance annuities

Provisions in the UVG (compulsory accident insurance for employed persons) line of business are divided into

- provisions for claims that are not or not yet paid out as annuities, and
- provisions for claims for which annuities are paid.

This section contains a remark on the second category, namely annuity provisions.

UVG annuities consist of a basic annuity and a cost-of-living adjustment (TZ) (COLA), which is analogous to the inflation adjustment for AHV (State Old Age and Survivors' Insurance). The TZ COLA is funded by the interest surplus $\phi_{10/10} - z_{UVG}$, where z_{UVG} is the technical interest rate of 3.25% and $\phi_{10/10}$ the average of the last 10 ten-year spot rates. $\phi_{10/10}$ is calculated annually by the Federal Office of Public Health (FOPH) on the basis of the spot interest rates[C] published by the National Bank.

In principle, the spot interest rate means the zero coupon interest rate; however, it still includes interest rates of coupon-bearing Confederation bonds during an ongoing transition phase for the old year. The interest rate $\phi_{10/10}$ calculated for fiscal year 2005, for instance, is an average of average yields of the ten-year Confederation bonds for the years 1996 and 2000 and of the ten-year zero coupon interest rates for the years 2001 to 2005. The result is 3.12%; for the year before, it was 3.37%.

If the interest surplus does not suffice to pay the cost-of-living adjustment, then the UVG insurer may levy contributions from active UVG policyholders in its client base. However, a problem consists in the fact that it is not guaranteed that the individual UVG insurer has such a client base, in which case it may not be able to levy any contributions. This risk has been solved by the creation of the UVG Cost-of-Living Fund. This fund guarantees that a participating UVG insurer will receive an equalization payment from the pool. Membership in this fund is currently (2006) not compulsory, but with only a few exceptions, all UVG insurers participate in the fund.

Since the effective liability of a UVG insurer depends on whether it is a member of the pool or not, the valuation must take this distinction into account.

[C] The spot interest rates are available from the Swiss National Bank at www.snb.ch → Publications → Monthly Statistical Bulletin → E Interest rates and yields → Yields on bonds.

3.3.1.1. Best estimate provision for a non-member

The best estimate provision for a non-member is the cash value of an indexed annuity. The annual payment of the annuity without cost-of-living adjustment shall have the value a. The payment in year i is again a, but corrected for inflation $(1+t_i)^i$, i.e., $a \cdot (1+t_i)^i$. The cash value of the payment flow of the indexed annuity payments is therefore

$$PV = \sum_{i=1} \frac{a \cdot (1+t_i)^i}{(1+r_i^{(0)})^i} \approx \sum_{i=1} \frac{a}{(1+r_i^{(0)}-t_i)^i}.$$

$r_i^{(0)}$ denotes the i-year, risk-free interest rate at time t_0. As a simplification, it is assumed that the difference $r_i^{(0)} - t_i$ between the current value of the i-year interest rate and the inflation rate can be approximated with a real interest rate, which is assumed to be 1.5%.
This entails:

$$PV \approx \sum_i \frac{a}{(1+0.015)^i}.$$

The UVG annuity provisions consist of this value and the provisions according to UVV (Accident Insurance Ordinance) 111/3, since they often have the meaning of required retirement provisions. Provisions according to UVV 111/1 are, however, not considered risk-bearing. In a catastrophic event, the UVG insurer could dissolve them, hence they are not part of the best estimate of the provisions.

This value must be increased by the value of the provisions.

3.3.1.2. Best estimate provision for a member of the Cost-of-Living Fund

The SST assumes that the UVG Cost-of-Living Fund will continue to exist and function in the future. This entails that a pool member can count on receiving contributions (equalization payments from the pool) for funding inflation if this should become necessary. This means that the annuity provision need not contain the future cost-of-living adjustments.

The valuation of the UVG annuity provisions for an insurer participating in the pool is composed of

- the annuity coverage capital, based on the rule in UVV 108,
- the obligations relating to the Cost-of-Living Fund, and
- the provisions under UVV 111/3.

4. Standard model for insurance, market, and credit risks

4.1. Standard model for market risks (without credit risk)

Please refer to the documents

- "SST 2006 Market Risk Model" and
- "Description of the input for the sensitivities in the market risk model for the SST Field Test 2006".

The market risk model in the standard model is based on the assumption that the change of the risk-bearing capital due to market risks can be described as a dependency on market risk factors. These market risk factors encompass interest rates over different terms and currencies, stock indices, currency exchange rates, real estate indices, bond spreads, implicit volatilities, etc. In total, the SST Field Test 2006 examines 74 market risk factors.

Furthermore, the standard model assumes that the market risk factors have a multivariate normal distribution. For most of the factors, the volatilities and correlation coefficients are given. Exceptions exist, for example, with respect to the volatilities and dependencies of hedge funds and investments in private equality. Different hedge funds and private equity behave very differently, which is why it is inappropriate to set fixed values for these risk factors. Instead, they must be determined for the insurer's own portfolio.

Additionally, the sensitivities of the insurer's own portfolio must be identified. Sensitivities are the partial derivatives of the risk-bearing capital according to market risk factors. They are in general approximated by a difference quotient. This will be illustrated with an example:

For instance, the 10-year interest rate r_{10} in CHF is considered a risk factor. If it changes, then both assets and liabilities change, but generally not to the same extent. Accordingly, a risk with respect to the 10-year interest rate exists. In the example, an increase of r_{10} by 100 base points (bp) entails a reduction of the assets by CHF 1,000,000 and a reduction of the liabilities by CHF 1,200,000. The sensitivity of the RBC relative to r_{10} is therefore

$$s_{r_{10}} := \frac{\partial RTK}{\partial r_{10}} \approx \frac{-1-(1.2)}{100}\frac{\text{MCHF}}{\text{bp}} = \frac{200'000}{100}\frac{\text{CHF}}{\text{bp}} = 2'000\,\text{CHF}/\text{bp}\,.$$

...δRBC/δr_{10}...

The interpretation of this is that the RBC increases by CHF 2000 if the 10-year interest rate rises by one base point.

Accordingly, the variances and covariances of the risk factors and the dependencies of the assets and liabilities on the risk factors are known. This gives us the variance of the risk-bearing capital caused by the changes to the market risk factors:

$$Var = \begin{pmatrix} s_1\sigma_1 & \dots & s_{74}\sigma_{74} \end{pmatrix} \cdot \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \cdots & \rho_{1,74} \\ \rho_{2,1} & 1 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & 1 & \rho_{73,74} \\ \rho_{74,1} & \cdots & \cdots & \rho_{74,73} & 1 \end{pmatrix} \cdot \begin{pmatrix} s_1\sigma_1 \\ s_2\sigma_2 \\ \vdots \\ s_{73}\sigma_{73} \\ s_{74}\sigma_{74} \end{pmatrix}$$

In this equation, σ_i means the volatility of the market risk factor i, $\rho_{i,j}$ the correlation coefficient between the two market risk factors i and j, and s_i the sensitivity to the market risk factor i.

4.2. Standard model for credit risk: Capital adequacy requirements for credit risks under Basel II – Brief instructions for the SST

This section provides an overview of how the Basel II standard approach is to be applied in the SST for determining the capital adequacy requirements for credit risks. The references are to the paragraphs in the document "International Convergence of Capital Measurement and Capital Standards", June 2004, by the Basel Committee on Banking Supervision of the BIS[D].

Deviations from Basel II

- No capital adequacy requirements for shares and holdings (see 4.2.2.5 and 4.2.2.6)
- Recognition of pledged life insurance policies as collateral for reduction of credit risk (see 4.2.3.1)

4.2.1. Principles

All claims are weighted with a specific factor (risk weight) according to the external ratings of the counterparty/issuer. The product of the relevant exposure and the risk weight yields the "weighted risk asset".

The magnitude of the risk weight depends on the type of the counterparty or issuer (States, banks, enterprises, retail portfolios) and its external ranking (to the extent that one exists). Collateral and other forms of credit risk reduction lead to a reduction of the relevant exposure.

4.2.1.1. Ratings

Ratings by the rating agencies S&P, Moody's, and Fitch are recognized by the SST. Companies may request approval from FOPI to use the ratings of other rating agencies.

Depending on the type of the counterparty or the issuer (cf. §§ 53, 63, 66, 103), the ratings are mapped onto a risk weight.

For purposes of the SST, the ratings by Moody's and Fitch should be mapped onto S&P ratings according to the following table, and these should then be converted into risk weights according to the Basel II rules:

S&P	Moody's	Fitch
AAA	Aaa	AAA
AA-	Aa3	AA-
A+	A1	A+
A-	A3	A-
BBB+	Baa1	BBB+
BBB-	Baa3	BBB-
BB+	Ba1	BB+
BB-	Ba3	BB-
B-	B3	B-
unrated	unrated	unrated

If other ratings are used than those by S&P, Moody's, and Fitch, the request should be accompanied by a mapping index according to the schema above.

[D] http://www.bis.org/publ/bcbs107.htm

Companies may use a subset of the rating agencies mentioned above and the additionally approved rating agencies. This subset must be clearly defined, and if more than one rating agency is used, then §§ 96-98 must be taken into account in determining the risk weight.

Issuer ratings and issues ratings must also be distinguished; see §§ 99-101.

4.2.1.2. Type of counterparty or issuer

The Basel II framework distinguishes different types of counterparties or issuers:

- States and their central banks, State organizations, other public offices, and multilateral development banks (§§ 53-59)
- Banks (§§ 60-64)
- Securities firms (§ 65)
- Corporates (§§ 66-68)
- Retail portfolios (§§ 69 – 71)
- Claims secured by residential property (§ 72)
- Claims secured by commercial real estate (§ 74)

For States, State organizations, other public offices (§ 53), banks, securities firms (§ 63), and corporates (§ 66), tables with risk weights are defined that reflect the risk weight as a function of the external ratings for the counterparty or the issuer.

Special items such as past due loans (§§ 75 – 78), higher-risk categories (§§ 79 – 80) and off-balance sheet items (§§ 82 – 89) are regulated separately.

4.2.1.3. Weighted risk assets

The net exposure is multiplied by the risk weight, which depends on the type of the counterparty or the issuer and its rating, resulting in a risk-weighted asset. Credit risk mitigation techniques (CRM, see section 4.2.3) lead to an adjustment of the risk weights – at least within the scope of the simplified approach.

When allowing credit risk mitigation by means of the comprehensive approach, the relevant exposure is derived from the gross exposure, reduced by the effect of any collateral.

The relevant exposure of derivatives and contingent liabilities is calculated according to 4.2.4 and 4.2.5.

4.2.1.4. Aggregation

The risk aggregation under Basel II is purely additive, i.e. portfolio and diversification aspects are already taken into account in the provided risk weights.

The total of the risk-weighted assets corresponds to the sum of the individual risk-weighted assets.

4.2.1.5. Capital adequacy requirements

The capital adequacy requirements for credit risks amount to 8% of the sum of all weighted risk assets.

4.2.2. Receivables

4.2.2.1. Bonds

Bond portfolios should be treated as receivables relating to the issuer, i.e., they should be weighted with the weights for States, banks, corporates, etc. depending on the type of issuer.

4.2.2.2. Loans

Loans, with the exception of mortgages fulfilling the requirements of § 72, should be treated as receivables relating to debtors, i.e. they should be weighted with the weights for States, banks, corporates, etc. depending on the type of debtor.

Receivables secured by commercial real estate are treated according to § 74.

4.2.2.3. Mortgages

Mortgages fulfilling § 72 are weighted at 35%.

4.2.2.4. Off-balance sheet items

Off-balance sheet items encompass several item types such as derivatives, guarantees, and loan commitments. All off-balance sheet items have in common that their amounts are converted into a relevant exposure with the use of credit conversion factors (CCF) (§§ 82 – 89). The CCF represents the potential future risk exposure.

The relevant exposures determined in this way are then multiplied by risk weights depending on the type of the counterparty (see section 4.2.2.2), thereby converting them into a weighted risk asset.

<u>Derivatives</u>

Derivative positions can result in a counterparty risk. The treatment of derivatives not traded on a recognized exchange and not subject to a daily margin call is described in section 4.2.4.

<u>Guarantees</u>

The treatment of contingent liabilities and guarantees is explained in section 4.2.5.

<u>Loan commitments</u>

See § 83.

4.2.2.5. Shares

No capital adequacy requirements for credit risks.

4.2.2.6. Holdings

No capital adequacy requirements for credit risks.

4.2.2.7. Securitized receivables

§§ 538 – 605 regulate the treatment of securitized items.

4.2.3. Credit risk mitigation techniques

Credit risk mitigation techniques (CRM) encompass techniques for mitigating credit risks through collateral, guarantees, netting agreements, or credit derivatives. The effect of credit risk mitigation may (but does not have to) be taken into account in the SST.

Guarantees and credit derivatives can only be taken into account if they are direct, explicit, irrevocable, and unconditional (see §§ 140 – 141).

Credit risk mitigation can only be fully taken into account if the residual maturity of the exposure and the credit risk mitigation is identical (see § 143 and §§ 202 – 205).

Note: Receivables secured by commercial real estate are discussed in sections 4.2.2.2 and 4.2.2.3; the corresponding real security should not be taken into account with respect to credit risk mitigation.

4.2.3.1. Collateral

The SST offers two options for taking collateral into account: the simple approach and the comprehensive approach.

Simple approach

In the simple approach according to §§ 182 – 185, the risk weight of the exposure is replaced by the risk weight of the credit risk mitigation. § 145 describes the collateral instruments that may be taken into account.
In addition to the collateral instruments described in § 145, a pledged life insurance policy may be taken into account as collateral up to the surrender value. If the creditor of the claim is also the issuer of the policy, then the proportion of the claim secured by the policy receives a risk weight of 0% (supplement to §§ 183 – 185).

Comprehensive approach

The comprehensive approach offers a more detailed consideration of collateral and permits additional collateral instruments to be taken into account in accordance with § 146. In the comprehensive approach, the volatility of the secured proportion is calculated with haircuts, which should be taken into account with respect to both exposure and collateral (see §§ 151 – 153). The relevant exposure is calculated according to the formula in § 147.

Insurance companies may use their own haircuts. They must show that all conditions in accordance with §§ 154 – 181 are met.

4.2.3.2. Guarantees

In the case of guarantees meeting the conditions of §§ 189 – 190 and § 195, the protected part of the original exposure is weighted with the risk weight of the protection provider (see § 196 – 201).

4.2.3.3. Netting agreements

The risk-mitigating aspect of netting agreements should be taken into account in accordance with § 188.

4.2.3.4. Credit derivatives

Only CDS and TRS may be taken into account as credit risk mitigation techniques in the SST (see §§ 193 – 194). If the conditions of §§ 189 – 192 and § 195 are met, then the protected portion of the original exposure is weighted with the risk weight of the protection provider (see §§ 196 – 201).

4.2.4. Credit exposures of derivatives

In the case of forward contracts (including non-balanced, non-fulfilled spot transactions), the credit equivalent can either be calculated according to the market valuation method or the original risk method. In the case of purchased options, the market valuation method must always be used.

4.2.4.1. Market valuation method

When using the market valuation method, the relevant exposure is calculated on the basis of *the current replacement value* of the contract in question, plus an add-on to cover the future potential credit risk during the residual maturity of the contract. An add-on may be offset up to its amount with the negative replacement value of the contract in question.

The following add-ons (in percent) apply to forward contracts and purchased options, by underlying instrument:

	< 1 year maturity	**1-5 years maturity**	**> 5 years maturity**
Interest rates	0.0	0.5	1.5
Foreign currency and gold	1.0	5.0	7.5
Shares	6.0	8.0	10.0
Share indices	4.0	5.0	7.5
Precious medals	7.0	8.0	10.0
Other basic commodities	12.0	13.0	15.0

The maturity of the underlying instrument is used for interest rate contracts, and the maturity of the contract is used for other instruments.

4.2.4.2. Original risk method

When using the original risk method, the relevant exposure is calculated by multiplying *the nominal value* of the contract in question by its credit conversion factor.

The following credit conversion factors (in percent) apply to forward contracts and purchased options, by underlying instrument:

	Original maturity: **1 year**	**For every other year begun**
Interest rates	1.0	2.0 p.a.
Foreign currency and gold	4.0	6.0 p.a.
Shares	12.0	9.0 p.a.
Share indices	8.0	6.0 p.a.
Precious medals	14.0	10.0 p.a.
Other basic commodities	24.0	18.0 p.a.

4.2.4.3. Basis of calculation

Add-ons and credit conversion factors are calculated on the following basis:

- For instruments such as forward rate agreements, interest rate swaps, and the like, on the basis of the nominal value of the contract or the cash value of the receivables-side consisting of nominal value and interest;
- For currency swaps, on the basis of the nominal value of the receivables-side, i.e. the basis of calculation applicable to the determination of the received interest payment, or on the basis of the cash value of the receivables-side consisting of nominal value and interest;
- For share index swaps, precious metal swaps, nonferrous metal swaps, and commodity swaps, on the basis of the agreed nominal remuneration or – if no nominal remuneration has been agreed – on the basis of the "amount X fixed price" or the market value of the performance claim or the cash value of the receivables-side consisting of nominal value and interest;
- For other forward transactions, on the basis of the market value of the money claim or the performance claim;
- For options, analogous to other forward transactions, but with appropriate delta weighting.

4.2.4.4. Exceptions

An add-on can be omitted in the case of:

- Contracts with an original maturity of at most 14 calendar days;
- *Contracts traded on a recognized exchange* where they are *subject to a daily margin call*, with the exception of purchased options;
- Contracts traded off-exchange that meet all of the following conditions:
- the contracts are traded on a representative market;
- the transactions are made on a covered basis; the cover consists of cash deposits or pledged or at least equivalently protected tradable instruments, precious metals, and commodities;
- the contracts and the cover are valued daily at market prices and are subject to a daily margin equalization.

4.2.4.5. Netting agreements

Companies using the market valuation method may offset positive replacement values and all add-ons as well as negative replacement values with forward contracts and options with the same counterparty, as long as a bilateral agreement with this counterparty exists that is shown to be recognized and enforceable under the following legal orders:

- the law of the State in which the counterparty is domiciled and, if a foreign branch establishment of an enterprise is involved, additionally the law of the domicile of the branch establishment; and
- the law relevant to the individual transactions included; and
- the law to which the agreements are subject that are required to effect the offset.

The offset is permissible in the following cases:

- for all transactions included in a netting agreement according to which the bank, in the case of default of the counterparty due to insolvency, bankruptcy, liquidation, or similar circumstances, only has the right to receipt or only the obligation to pay the difference between the non-realized gains and losses from the included transactions (close-out netting); or
- for all reciprocal claims and obligations due the same day in the same currency that have been combined by a debt conversion agreement between the bank and the counterparty in such a way that this debt conversion results in a single net amount and thereby creates a new, legally binding contract that cancels the previous contracts (netting by novation); or
- for squared transactions, as long as a payment-netting agreement exists, in accordance with which the reciprocal payment obligations on the due date are determined for each currency on a balance basis and only this balance amount is paid.

The offset is impermissible if the agreement contains a provision allowing the non-defaulting party to only make limited payments or no payments to the defaulting party, even if the latter is a creditor according to the balance (walk-away clause).

4.2.5. Contingent liabilities

In the case of contingent liabilities and irrevocable commitments, the relevant exposure is calculated by multiplying the nominal value or the cash value of the transaction in question with its credit conversion factor.

The following credit conversion factors apply:

Factor	Instruments
0.5	• Guarantees such as bid bonds, performance bonds, including construction sureties that should not be weighted with the factor 0.25; • Other guarantees such as aval, surety, and guarantee commitments as well as other commitments from standby letters of credit that are not used to cover the del credere risk; • Unsecured irrevocable loan commitments that have not been used, including note issuance facilities, revolving underwriting facilities, and similar instruments with a fixed commitment of over one year residual maturity; • Performance-related advance guarantees;
1.0	• Aval, surety, and guarantee commitments as irrevocable standby letters of credit used to cover the del credere risk;
1.25	• Payment and subsequent payment commitments on shares and other participation instruments not balanced under holdings;
2.5	• Payment and subsequent payment commitments on shares and other participation instruments if they do not relate to consolidated holdings;
6.25	• Payment and subsequent payment commitments on shares and other participation instruments if they relate to consolidated holdings.

Contingent liabilities of which the insurance has transferred sub-holdings can be weighted to the extent of the sub-holdings as direct receivables relating to the sub-holdings in question.

4.3. Standard model for life insurance

4.4. Standard model for non-life and accident insurance

Section 4.4 presents the standard model for non-life and accident insurers. First, additional notations will be introduced in section 4.4.1 and then (section 4.4.2) some basic assumptions will be explained. Section 4.4.3 discusses the classification of insurance into different lines of business.

Finally, sections 4.4.4 and 4.4.5 will examine what the definition of target capital in (2b)

$$ZK_\alpha = -ES_\alpha\left(\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0)\right) + \frac{MvM(1)}{1+r_1^{(0)}}.$$

TC_α = ...(RBC(1)/... – RBC(0)) + ...

means for a non-life and accident insurer. The term $\frac{MvM(1)}{1+r_0}$ will, however, only be described in section 6, so that we will restrict ourselves here to the first term $ES_\alpha\left(\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0)\right)$ RTK->RBC. The goal is accordingly first of all to find a distribution for $\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0)$ RTK->RBC for which the expected shortfall can then be calculated at the certainty level 1-α. Since the risk-bearing capital is defined as the difference

$$RTK(t) = A(t) - L(t)$$

RBC (t) = ...
between the market value of all assets and the discounted best estimate of the liabilities, it holds that::

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) = \left(\frac{A(1)}{1+r_1^{(0)}} - A(0)\right) - \left(\frac{L(1)}{1+r_1^{(0)}} - L(0)\right). \tag{4}$$

RBC(1)... – RBC (0)...
This expression is expressed in the usual parameters and variables of non-life and accident insurance.

Sections 4.4.6 to 4.4.11 discuss how the distribution function for the arising stochastic and technical variables can be derived with respect to claims expenses and provisions.

4.4.1. Notations for non-life insurers

Claims date	Date to which a claim is assigned. In most lines of business, this is the occurrence date of the claim. Exceptions are in lines of business in which "claims made" policies are the rule. In such cases, the claims date is the notification date of the claim.
CY	Abbreviation for "Current Year", i.e. the calendar year in which the SST is conducted.
CY claims, new claims	Claims whose claims date is in the CY. From the perspective of 1 January of the CY, these claims lie in the future and are therefore called new claims.
PY	Abbreviation for "Previous Years". These are the years preceding the year in which the SST is conducted.
PY claims	Claims whose claims date is in the PY.

t_0	Beginning of the CY
t_1	End of the CY
upr	Unearned premium reserve on 1 January of the CY
P	Estimate at time t_0 for the earned premiums in the CY (deterministic quantity)
K	Estimate at time t_0 for administrative and operational costs in the CY (deterministic)
S_{CY}	Random variable for the undiscounted claims expense of the CY
S_{CY}^{GS} S^{MC}_{CY}	Contribution of major claims to S_{CY}.
S_{CY}^{NS} S^{NC}_{CY}	Contribution of normal claims (minor claims) to S_{CY}.
$(\alpha_k)_{k\geq 0}$	Payment pattern for the CY claims, normalized to $\sum_{k\geq 0}\alpha_k = 1$. By convention, it is assumed that claims payments are made at the end of each year. The index $k = 0,1,2,...$ numbers the payment year. The payment at the end of the CY is therefore given by $\alpha_0 S_{CY}$.
$R_{py}^{(0)}$	Best estimate of the claims provisions on 1 January of the CY for PY claims.
$(\beta_k)_{k\geq 0}$	Payment pattern for the PY claims, normalized to $\sum_{k\geq 0}\beta_k = 1$; k denotes the payment year, where $k = 0$ refers to the current year.
$C_{py}\times R_{py}^{(0)}$	New assessment of the expenditure $R_{py}^{(0)}$ on 31 December of the CY, i.e. new assessment of the payment in the CY and the initial provisions of PY claims on 31 December of the CY. C_{py} serves as a stochastic correction factor. $(1-C_{PY})R_{PY}^{(0)}$ is therefore the undiscounted settlement result.
D and d	Discount factors for claims, defined as the relationship between the discounted value and the nominal value of an observed set of claims.
$D_{PY}^{(1)}$	Random variable for the discount factor on 31 December of the CY for the PY claims, defined by $D_{PY}^{(1)} = V_0^{(1)}\cdot\beta_0 + V_1^{(1)}\cdot\beta_1 + ... + V_n^{(1)}\cdot\beta_n$. The discount factor depends on the interest-rate curve on 31 December CY (which is uncertain from the perspective of 1 January CY) and the payment pattern for PY claims.
$D_{CY}^{(1)}$	Random variable for the discount factor on 31 December CY for the CY claims, defined by $D_{CY}^{(1)} = V_0^{(1)}\cdot\alpha_0 + V_1^{(1)}\cdot\alpha_1 + ... + V_n^{(1)}\cdot\alpha_n$. The discount factor depends on the interest-rate curve on 31 December CY (which is uncertain from the perspective of 1 January CY) and the payment pattern for CY claims.
$d_{CY}^{(0)}$	Discount factor on 1 January CY with the interest-rate curve of 1 January CY (for CY claims), defined by $d_{CY}^{(0)} = v_1^{(0)}\cdot\alpha_0 + v_2^{(0)}\cdot\alpha_1 + ... + v_{n+1}^{(0)}\cdot\alpha_n$.

$d_{PY}^{(0)}$	Discount factor on 1 January CY with the interest-rate curve of 1 January CY (for PY claims), defined by $$d_{PY}^{(0)} = v_1^{(0)} \cdot \beta_0 + v_2^{(0)} \cdot \beta_1 + ... + v_{n+1}^{(0)} \cdot \beta_n .$$
α	has several meanings. First, it designates the quantile level for the SST, usually with the value 1%. Second, α was used above as a symbol for payment patters for claims that have already occurred, and third, α also serves as a designation for the Pareto parameters of the major claims distributions.
NHP	Natural hazard pool
BI	Business interruption
MVL	Motor vehicle, liability
MVC	Motor vehicle, comprehensive

4.4.2. Basic assumptions

The standard model of the SST for non-life insurers starts with the following basic assumptions:

- Risk arises from uncertainties:
  1. in the investments (value fluctuations and default) and in the future interest-rate curve with simultaneous effects on assets and liabilities,
  2. in the claims expense for new claims (CY claims), and
  3. in the amount of the claims provisions.
- The following are considered deterministic:
  1. the earned premiums P for the current year (CY),
  2. the operational and administrative costs K,
  3. the settlement patterns $(\alpha_k)_{k\geq 0}$ and $(\beta_k)_{k\geq 0}$ for CY and PY claims. (Not only the amount, but also the settlement speed of the provisions is considered to be stochastic.)
- The randomness of the future interest rates is dependent on the technical variables such as the amount of the claims or nominal claims provisions.
- The undiscounted claims provisions for PY claims are such that the expected value is neither a settlement gain nor a settlement loss. In other words, the expected value of the settlement result is zero, and the provisions are determined according to the best estimate. For the correction factor introduced above, this means: $E[C_{PY}] = 1$.

By convention, premiums and costs are transacted at the beginning of the year, and claims payments at the end of the year. No new business is taken into account that arises after the end of the current year. Any initial premium transfer is therefore not taken into account.

The costs are differentiated according to

- Claims processing costs, i.e. costs that are related to the processing of claims. Further terminological distinctions are:
  1. Non-allocable claims processing costs. These are costs related to claims processing that cannot be allocated to an individual claim, such as the salaries of employees, the maintenance of IT systems, and other claims administration costs. Often, the abbreviation ULAE = "unallocated loss adjustment expenses" is used.
  2. Allocable claims processing costs. These are costs that can be allocated to a particular claim, such as court costs, costs for external lawyers, etc.
- Operational and administrative costs K.

Provisions must be set aside for future claims processing costs (ULAE and ALAE) for claims with a claims date in the past. Often, ALAE provisions are already included in the claims provisions. In this case, only the ULAW provisions must be considered separately. One possibility for this is to use the "New York" method.

4.4.3. Classification of lines of business (LoB) in the SST

13 lines of insurance business are considered, which are listed in appendix 8.4.1 While the settlement risks (risk in the provisions for PY claims) are considered according to these lines of business, the consideration of new claims risks looks at natural hazard claims in the property business separately. The natural hazard claims are composed of claims in the natural hazard pool (NHP) and other natural hazards, such as business interruption losses triggered by a natural hazard event. The reason is that the NHP claims and the other natural hazard claims are closely correlated with respect to major claims. In the SST standard model, they are even considered co-monotone.

4.4.4. Separation of the total risk into technical risks and financial market and ALM risks

The claims risk for an insurance company consists both in a market risk and a technical risk, since the discounted value of the liabilities depends both on the interest rates and on the nominal value of the claim. The total risk is expressed by a multiplication of interest rate risks and insurance risks. Section 4.4.4 shows how the two risk components must be decoupled by linearizing the mathematical product. However, it must be pointed out that the linearization entails a simplification (1st order Taylor development of the product of two independent variables at the point of the expected value of the two variables).

The result is a sum of terms of which one component can be identified with the market risk and another component can be identified with the technical risk.

4.4.4.1. CY claims

For purposes of introducing the consideration of CY claims, we will ignore provisions for PY claims and assume that we are starting with a "clean slate". In section 4.4.4.1, we will therefore examine formula (4) only for claims from the current year CY.

Furthermore, we will start with assets with the value A(0) at the beginning of the calculation.
As liabilities for CY claims, only the reserves for premiums transfers are available at the beginning, so that L(0) = upr.

Since the SST does not observe nominal values, but rather cash values, we must specify in the following at what times the payments flow. For this purpose, we will assume that the earned premiums P and the administrative costs K are received and paid at the beginning of the year. Over the course of the year, the resulting amount $A(0)+(P-upr)-K$ generates a stochastic return R_I.
New claims (CY claims) arise over the course of the year. Their sum is equal to the undiscounted annual claims expense S_{CY}.
Finally, claims payments are made for the new claims at the end of the year. According to the definition of the payment pattern for CY claims, these claims payments equal $\alpha_0 \cdot S_{CY}$.

Therefore, the value of the assets at the end of the year equals

$$A(1) = (A(0) + (P - upr) - K)(1 + R_I) - \alpha_0 S_{CY}. \qquad (5)$$

The liabilities at the end of the current year (i.e. at time t_1) consist of the cash value of the future claims payments ($t_2, t_3, t_4, ...$) for claims from the event year CY and can therefore be calculated as

$$L(1) = (V_1^{(1)} \cdot \alpha_1 + V_2^{(1)} \cdot \alpha_2 + ... + V_n^{(1)} \cdot \alpha_n) \cdot S_{CY} . \tag{6}$$

Using formula (4), the total discounted change of RBC for the CY is:

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) = \tag{7}$$

$$\frac{1}{1+r_1^{(0)}} \left[(A(0) - upr + P - K) \cdot (1 + R_I) - D_{CY}^{(1)} \cdot S_{CY} - (1 + r_1^{(0)}) \cdot (A(0) - upr) \right],$$

RBC (1)... – RBC (0)...
where $D_{CY}^{(1)}$ is the discount factor for the CY claims and is defined as

$$D_{CY}^{(1)} \cdot S_{CY} = (V_0^{(1)} \cdot \alpha_0 + V_1^{(1)} \cdot \alpha_1 + ... + V_n^{(1)} \cdot \alpha_n) \cdot S_{CY} . \tag{8}$$

Both the discount factors $V_j^{(1)}$ and the claims expense S_{CY} are stochastic quantities. We are therefore dealing with a product of random variables, where the first is related to the ALM risk, and the second to the pure insurance risk. To separate the two risk contributions, we must perform the following linearization of formula (8):

$$D_{CY}^{(1)} \cdot S_{CY} \approx \tag{9}$$

$$E[D_{CY}^{(1)}] \cdot E[S_{CY}] + \left(D_{CY}^{(1)} - E[D_{CY}^{(1)}]\right) \cdot E[S_{CY}] + E[D_{CY}^{(1)}] \cdot \left(S_{CY} - E[S_{CY}]\right).$$

The first term is the product of the expected discount effect and the expected claims expense. The second term describes the effect of the interest rate uncertainty on the expected claims expense, and the third term describes the variability of the claims expense given a fixed, expected discount factor (i.e. the given interest rates). Formula (7) can therefore be redefined as

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) \approx$$

RBC (1)... RBC (0)...

$$\frac{1}{1+r_1^{(0)}} \Big((R_I - E[R_I]) \cdot (A(0) - upr + P - K) - \left(D_{CY}^{(1)} - E\left[D_{CY}^{(1)}\right]\right) \cdot E[S_{CY}]$$

$$+ \left(E[R_I] - r_1^{(0)}\right) \cdot (A(0) - upr + P - K) + r_1^{(0)} \cdot (P - K) \tag{10}$$

$$+ (P - K) - E[D_{CY}^{(1)}] \cdot E[S_{CY}]$$

$$- E[D_{CY}^{(1)}] \cdot \left(S_{CY} - E[S_{CY}]\right)\Big). .$$

This formula can be interpreted as follows:
The change of the discounted RBC for the CY claims is composed of the financial risk and the ALM risk (line 1), the expected capital returns on the assets (line 2), the expected technical result (line 3), and the technical risk (line 4).

4.4.4.2. PY claims

We now examine formula (4) for claims from event years prior to CY.

The best estimate (nominal value) of the provisions at time t_0 shall be designated $R_{PY}^{(0)}$. The discounted value is

$$L_{PY}^{(0)} = \sum_{j=0} v_{i+1}^{(0)} \beta_i R_{PY}^{(0)} = d_{PY}^{(0)} R_{PY}^{(0)}, \tag{11}$$

where $d_{PY}^{(0)}$ is the discount factor defined by the equation above.
The value of the assets at time t_0 shall be $A(0)$.
The information gains during the year result in a correction of the best estimate. Shortly before the payment is made at the end of the year, the estimate of the future payments has been corrected by the factor C_{PY}. The new estimate is therefore $C_{PY} \cdot R_{PY}^{(0)}$.
The payment made at the end of the year (time t_1) is $\beta_0 \cdot C_{PY} R_{PY}^{(0)}$, and the future payments will accordingly be $\beta i \cdot C_{PY} R_{PY}^{(0)}$, $i = 1,2,3,\ldots$.
For the discounted best estimate at time t_1 (after the payment), this entails:

$$L_{PY}^{(1)} = \sum_{i=1} V_i^{(1)} \cdot \beta_i \cdot C_{PY} R_{PY}^{(0)} \tag{12}$$

At the end of the year, the nominal value of the assets has decreased by the value of the payment, $\beta_0 \cdot C_{PY} R_{PY}^{(0)}$. Substitution in formula (4) therefore gives us

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) = -\frac{\beta_0 \cdot C_{PY} R_{PY}^{(0)} + \sum_{i=1} V_i^{(1)} \cdot \beta_i \cdot C_{PY} R_{PY}^{(0)}}{1+r_1^{(0)}} + d_{PY}^{(0)} R_{PY}^{(0)} \tag{13}$$

RBC(1)... RBC (0)...
We rewrite the numerator in the first term as

$$\sum_{j=0} V_i^{(1)} \cdot \beta_i \cdot C_{PY} R_{PY}^{(0)} = D_{PY}^{(1)} \cdot C_{PY} R_{PY}^{(0)} \tag{14}$$

As in the case of CY claims, the risk of the provisions consists of a product of the interest rate risk (expressed by the discount factor $D_{PY}^{(1)}$) and through a change of the nominal provisions (expressed by the correction factor C_{PY}). As in the case of CY risks, this product can be linearized as follows:

$$\begin{aligned} D_{PY}^{(1)} \cdot C_{PY} &\approx E[D_{PY}^{(1)}] \cdot E[C_{PY}] + E[D_{PY}^{(1)}] \cdot (C_{PY} - E[C_{PY}]) + (D_{PY}^{(1)} - E[D_{PY}^{(1)}]) \cdot E[C_{PY}] \\ &= E[D_{PY}^{(1)}] + E[D_{PY}^{(1)}] \cdot (C_{PY} - 1) + D_{PY}^{(1)} - E[D_{PY}^{(1)}] \\ &= E[D_{PY}^{(1)}] \cdot (C_{PY} - 1) + D_{PY}^{(1)} \end{aligned} \tag{15}$$

where we have made use of $E[C_{PY}] = 1$.
Substitution results in

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) \approx -\frac{D_{PY}^{(1)}R_{PY}^{(0)} + E[D_{PY}^{(1)}](C_{PY}-1)R_{PY}^{(0)}}{1+r_1^{(0)}} + d_{PY}^{(0)}R_{PY}^{(0)}$$

$$= -\frac{D_{PY}^{(1)} - E[D_{PY}^{(1)}]}{1+r_1^{(0)}}R_{PY}^{(0)} - \left(\frac{E\left[D_{PY}^{(1)}\right]}{1+r_1^{(0)}}C_{PY} - d_{PY}^{(0)}\right)R_{PY}^{(0)} \tag{16}$$

RBC(1)... RBC (0)...
The first term on the right side describes the interest rate risk, and the second term is the risk of changes to the nominal amount of the provisions.

4.4.4.3. PY and CY claims

If we combine the results for CY and PY claims, we receive the following key formula (17a):

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) \approx$$

RBC (1)... RBC (0)...

$$\frac{R_I - E[R_I]}{1+r_1^{(0)}}\cdot(A(0)+(P-UPR)-K) - \frac{D_{CY}^{(1)} - E[D_{CY}^{(1)}]}{1+r_1^{(0)}}\cdot E[S_{CY}] - \frac{D_{PY}^{(1)} - E[D_{PY}^{(1)}]}{1+r_1^{(0)}}\cdot R_{PY}^{(0)}$$

$$+\frac{E[R_I] - r_1^{(0)}}{1+r_1^{(0)}}\cdot(A(0)+(P-UPR)-K) \tag{17a}$$

$$+(P-K) - \frac{E\left[D_{CY}^{(1)}\right]}{1+r_1^{(0)}}\cdot E\left[S_{CY}\right]$$

$$-\frac{E\left[D_{CY}^{(1)}\right]}{1+r_1^{(0)}}\cdot\left(S_{CY} - E\left[S_{CY}\right]\right) - \left(\frac{E\left[D_{PY}^{(1)}\right]}{1+r_1^{(0)}}\cdot C_{PY} - d_{PY}^{(0)}\right)\cdot R_{PY}^{(0)}.$$

The four lines constituting the right side of the equation (17) have the following meaning and interpretation:

- The first line contains random quantities that reflect the financial market and ALM risk (market risk). The uncertain quantities in this line are the return R_I and the discount factors $D_{CY}^{(1)}$ and $D_{PY}^{(1)}$. These only appear as a difference from their expected values ($E[R_I]$, $E[D_{CY}^{(1)}]$, $E[D_{PY}^{(1)}]$). The ALM risk is modelled with a normal distribution.
- The second line captures the expected return on the assets above the risk-free one-year interest rate.
- The third line is the expected technical result.
- The fourth line is, like the first line, a random quantity. It contains the deviation of the technical result from its expected value. The uncertainties arise from the unknown claims expense for CY claims and the unknown settlement result from PY claims.

Lines 1 and 2 contribute to the financial market and ALM risk; their analysis (sensitivities) will be further discussed in section 2.7. For now, we will concentrate on the technical result and its risk (lines 3 and 4).

We will make two additional approximations for this purpose:

$$\frac{E\left[D_{CY}^{(1)}\right]}{1+r_1^{(0)}} \approx d_{CY}^{(0)}$$

and

$$\frac{E\left[D_{PY}^{(1)}\right]}{1+r_1^{(0)}} \approx d_{PY}^{(0)}. \tag{18}$$

The motivation for the two assumptions results from the following observation:
The left side of each equation contains the value of the expression $E\left[D_{CY}^{(1)}\right]$ or $E\left[D_{PY}^{(1)}\right]$ discounted to time t_0, i.e. the term is discounted with the interest-rate curve current at time t_1 to t_1 and then with $1+r_1^{(0)}$ to t_0. The right side contains the discount factor, which discounts directly to t_0 based on the interest-rate curve valid at time t_0.

The right side and the left side would agree if the interest-rate curve were flat and consistent over time ($r_k^{(0)} = r_{k+1}^{(0)} = E[R_k^{(1)}] = E[R_{k+1}^{(1)}] = r$), but also if the forward interest would correspond to the expected interest-rate curve on 31 December CY. However, this is not the case in general.

If the abovementioned approximation is made, the technical risk and the technical result can be simplified to

$$\left(P - K - d_{CY}^{(0)} E[S_{CY}]\right) - d_{CY}^{(0)}\left(S_{CY} - E[S_{CY}]\right) - d_{PY}^{(0)}\left(C_{PY} - 1\right) R_{PY}^{(0)}. \tag{19}$$

This is the key formula for modelling technical risks.

To calculate this term, the following quantities must be determined:

- estimate of the earned premium P and the costs K,
- estimate of the settlement patterns (α_i) and (β_i) and the resulting discount factors $d_{CY}^{(0)}$ and $d_{PY}^{(0)}$,
- estimate of the expected claims expense $E\left[S_{CY}\right]$ for CY claims
- probability distribution for the random variable S_{CY} of the CY claims
- probability distribution for the settlement result $(1 - C_{PY}) \cdot R_{PY}^{(0)}$

For sake of completeness, we will repeat formula (17a) with the approximation (18):

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) \approx$$

$$\approx \frac{R_I - E[R_I]}{1+r_1^{(0)}} \cdot (A(0) + (P - UPR) - K) - \left(\frac{D_{CY}^{(1)}}{1+r_1^{(0)}} - d_{CY}^{(0)}\right) \cdot E[S_{CY}] - \left(\frac{D_{PY}^{(1)}}{1+r_1^{(0)}} - d_{PY}^{(0)}\right) \cdot R_{PY}^{(0)}$$

$$+ \frac{E\left[R_I\right] - r_1^{(0)}}{1+r_1^{(0)}} \cdot (A(0) + (P - UPR) - K) \tag{17b}$$

$$+ P - K - d_{CY}^{(0)} \cdot E\left[S_{CY}\right]$$

$$- d_{CY}^{(0)} \cdot \left(S_{CY} - E\left[S_{CY}\right]\right) - d_{PY}^{(0)} \cdot \left(C_{PY} - 1\right) \cdot R_{PY}^{(0)}$$

RBC (1)... RBC (0)...
The task is to determine the distribution of the right side and ultimately to determine the expected shortfall.

It is useful to keep in mind that the changes to the RBC due to financial risks result from the sum of the second and third lines in (17b), and that the changes to the RBC due to technical variables are contained in the last two lines. It would therefore certainly be possible, when aggregating the right side of (17b), to first aggregate the second and third lines and the fourth and fifth lines, and then combine the two results.

The second possibility consists in aggregating the two stochastic lines (second and fifth) in a first step, and then calculating the expected shortfall from the result. This expected value of these two lines is zero. In a second step, the expected financial result and the expected technical result can then be added together.

In section 4.4.6, we will discuss the determination of the probability distributions of the random variables S_{CY} and $(1-C_{PY})\cdot R_{PY}^{(0)}$ in more detail. First, we will make a few remarks on the assumptions made in (18).

4.4.4.4. Accident insurance annuities

In section 3.3.1, we considered the valuation of provisions for accident insurance (UVG) annuities separately. Now, we will look at what the term $RTK(1)/(1+r_1^{(0)})-RTK(0)$ RBC(1)... RBC(0)... means for such annuities.
In the case of a UVG insurer that is not a member of the Cost-of-Living Fund, no additional remarks are necessary here. For the more frequent case that an insurer participates in the Cost-of-Living Fund, the following considerations arise.

The annuity provisions are composed of (see section 3.3.1.2) the coverage capital DK CC and the liabilities TF COLF relating to the Cost-of-Living Fund, as well as the provisions in accordance with the Accident Insurance Ordinance (UVV 111/3) which will not be considered here further:

$$L = DK + TF + UVV_{111/3} \quad \text{L = CC + COLF + UVV}_{111/3}$$

<u>Coverage capital</u>

The coverage capital consists of the sum of the future payments c_j^{UVG} for the basic annuities.
The SST makes the simplifying assumption with respect to payment times that the payments take place at the end of the year, the end of the following year, etc. The coverage capital for the existing basic annuities is therefore

$$DK_0 = \frac{c_1^{UVG}}{1+z} + \frac{c_2^{UVG}}{(1+z)^2} + \frac{c_3^{UVG}}{(1+z)^3} + \dots \quad \text{CC}_0 = \dots$$

and

$$DK_1 = \frac{c_2^{UVG}}{1+z} + \frac{c_3^{UVG}}{(1+z)^2} + \dots \quad \text{CC}_0 = \dots$$

for times t_0 and t_1. The payment c_1^{UVG} is no longer contained in DK_1 CC_1. It immediately follows that DK_0 CC_0 and DK_1 CC_1 are related by

$$DK_1 = (1+z)DK_0 - c_1^{UVG}$$ CC₁ = ... CC₀...

I.e., the new CC is the old CC, increased by the technical interest rate, minus the payment in the current year.

Moreover, we currently assume that the payments c_j^{UVG} for the basic annuities can be modelled as deterministic quantities. In other word, no biometric risks need to be considered.

Cost-of-living adjustment

In addition to c_1^{UVG}, a cost-of-living adjustment TZ COLA must be paid out to the beneficiaries. The SST assumes that the cost-of-living adjustment also becomes due at the end of the year.

Cost-of-Living Fund

The liabilities relating to the Cost-of-Living Fund at time t_0 (beginning of the year) shall be TF_0 $COLF_0$. During the year, the liabilities are increased. The increase is composed of the interest payments on the already existing liability ($\phi_{10/10} \cdot TF_0$ $COLF_0$), the interest rate difference $(\phi_{10/10} - z)$ on the coverage capital DK_0 CC_0, and any received contribution premium P_{UmLage} $P_{contrib}$ from the active lives. $\phi_{10/10}$ is an interest rate introduced in section 3.3.1. A reduction of the liabilities relating to the pool arises in the amount of the cost-of-living adjustment paid to the beneficiaries. The addition results in a liability relating to the Cost-of-Living Fund at the end of the year in the amount of

$$TF_1 = (1+\phi_{10/10}) \cdot TF_0 + (\phi_{10/10} - z) \cdot DK_0 + P_{Umlage} - TZ\,.$$ $COLF_1 = ...\ COLF_0+...CC_0+P_{contrib}-COLA$

Resulting total liabilities

The value of the provisions and the end of the year is (not taking UVV 111 into account):

$$\begin{aligned} L_1 &= DK_1 + TF_1 \\ &= (1+z)DK_0 - c_1^{UVG} + (1+\phi_{10/10}) \cdot TF_0 + (\phi_{10/10} - z) \cdot DK_0 + P_{Umlage} - TZ \\ &= (1+\phi_{10/10})(DK_0 + TF_0) + P_{Umlage} - TZ - c_1^{UVG} \\ &= (1+\phi_{10/10})L_0 + P_{Umlage} - TZ - c_1^{UVG}\,. \end{aligned}$$

DK -> CC, TF -> COLF, TZ -> COLA, P_{Umlage} -> $P_{contrib}$

Assets

At times t_0 and t_1, the assets have the value A_0 and

$$A_1 = (1+R) \cdot A_0 + P_{Umlage} - c_1^{UVG} - TZ\,.$$

P_{Umlage} -> $P_{contrib}$, TZ -> COLA

Any contribution premium P_{Umlage} $P_{contrib}$ is paid by the active policyholders to the insurer, and the payments c_1^{UVG} and TZ COLA are paid to the beneficiaries. $R \cdot A_0$ is the performance of the assets invested during the year.

Change to risk-bearing capital

Substituting the expressions of assets and liabilities in $RTK(1)/(1+r_1^{(0)}) - RTK(0)$ RBC(1)...RBC(0)... results in

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) =$$

$$= \frac{\left((1+R)\cdot A_0 + P_{Umlage} - c_1^{UVG} - TZ\right) - \left((1+\phi_{10/10})(DK_0 + TF_0) + P_{Umlage} - TZ - c_1^{UVG}\right)}{1+r_1^{(0)}} - (A_0 - (DK_0 + TF_0))$$

$$= \frac{(R - r_1^{(0)})\cdot A_0 - (\phi_{10/10} - r_1^{(0)})\cdot(DK_0 + TF_0)}{1+r_1^{(0)}}$$

RTK -> RBC, P_{Umlage} -> $P_{contrib}$, TZ -> COLA, DK -> CC, TF -> COLF

It is remarkable that the terms for the conversion premium, the cost-of-living adjustment, and the technical interest rate no longer occur on the right side. The reason is the design of the Cost-of-Living Fund mechanism, especially the changes to the liabilities of insurers relating to the Fund.

The last line can be divided into an expected value and a stochastic component:

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) =$$

$$= \frac{(R - E[R] + E[R] - r_1^{(0)})\cdot A_0 - (\phi_{10/10} - E[\phi_{10/10}] + E[\phi_{10/10}] - r_1^{(0)})\cdot L_0}{1+r_1^{(0)}}$$

$$= \frac{1}{1+r_1^{(0)}}\cdot\left\{(E[R] - r_1^{(0)})\cdot A_0 + (R - E[R])\cdot A_0\right\}$$

$$- \frac{1}{1+r_1^{(0)}}\cdot\left\{(\phi_{10/10} - E[\phi_{10/10}])\cdot L_0\right\}$$

$$- \frac{1}{1+r_1^{(0)}}\cdot\left\{(E[\phi_{10/10}] - r_1^{(0)})\cdot L_0\right\}$$

RBC (1)... RBC(0)...

The first line represents the expected financial performance of the assets and the risk of the assets due to uncertainties in the financial risk factors. The significance and treatment of these terms is the same as in the preceding sections 4.4.4.1, 4.4.4.2 and 4.4.4.3.

The second line is the risk in the provisions arising from the uncertainty about the $\phi_{10/10}$ that is valid for the current year, but not known until the end of the year. For the 2006 test run, this uncertainty can be neglected.

The third line is the expected value of the change to the provisions. The expected value is non-zero if the expected value of $\phi_{10/10}$ is not equal to the one-year risk-free interest rate, which is generally the case. In view of the fact that $\phi_{10/10}$ was 3.37% and 3.12% in 2004 and 2005, respectively, and $r_1^{(0)}$ is currently about 1%, the quantity $(E[\phi_{10/10}] - r_1^{(0)})\cdot L_0$ cannot be neglected. $E[\phi_{10/10}]$ can be estimated as follows: When calculating the average over ten interest rates, the oldest, omitted interest rate of about 4% is replaced with a new interest rate of about 2%, which will lead to a reduction of

$\phi_{10/10}$ by about $\frac{1}{10} \cdot 2\% = 20bp$. For the 2006 test run, $E[\phi_{10/10}]$ can therefore be approximated with $3.12\% - 0.2\% \approx 2.9\%$.

It is interesting to look at the special case $A_0 = L_0$ (market value of the assets at the beginning is equal to the value of the liabilities at the beginning). Under these conditions, we have:

$$\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) = \frac{(R - r_1^{(0)}) \cdot L_0 - (\phi_{10/10} - r_1^{(0)}) \cdot L_0}{1+r_1^{(0)}}$$
$$= \frac{(R - \phi_{10/10}) \cdot L_0}{1+r_1^{(0)}}$$

RBC(1)...RBC(0)...

Even if we neglect the uncertainty in $\phi_{10/10}$, there is a risk that $\phi_{10/10}$ and R will not be equal. Even if the assets consist of ten tranches of risk-free ten-year zero coupon bonds, R will generally deviate from $\phi_{10/10}$, since the performance of the bond portfolio depends substantially on the change of the interest-rate curve over the year, whereas $\phi_{10/10}$ is a temporal average over the last ten years.
A market risk therefore exists. Essentially, liabilities behave like a bank account with an interest rate of $\phi_{10/10}$ (currently approximately the abovementioned 3%). However, there are no assets that generate a one-year risk-free performance of this magnitude. The asset portfolio consisting of the 10 tranches of risk-free bonds, for instance, generated a performance of about 2% in 2005.

On average, it may very well be possible for the asset portfolio consisting of 10 tranches of bonds to reach the performance of $\phi_{10/10}$ over a longer time period, but there is no guarantee in a single year.

> In summary, the current modelling (no biometric risks) indicates that UVG annuities do not entail a risk, but are associated with an expected loss in the amount of $(E[\phi_{10/10}] - r_1^{(0)}) \cdot L_0$.
> As for the rest of the SST, the risks of the assets are measured as the deviation of the performance R from the expected performance $E[R]$.

4.4.4.5. Comments on assumption (18) concerning the expected interest-rate curve

Assumption (18) posits the equality of

- the current (t_0) cash value of a payment, and
- the expected value of the value discounted to time t_1, after it has been subsequently discounted to time t_0.

The expected value enters the picture since the discounting to t_1 with the interest-rate curve takes place at the same time, and this interest-rate curve is not known at time t_0.

Accordingly, assumption (18) makes an assumption about the expected value of a future discount factor and therefore also about the expected value of future interest $R_i^{(1)}$:

$$E[\frac{1}{(1+R_i^{(1)})^i}] = \frac{1}{(1+r_1^{(0)}) \cdot (1+r_{j-1}^{(0)})^{j-1}}.$$

With this simplification, the claims provisions do not contribute to the technical result on a discounted basis, which is not clear *a priori*. It is true that the undiscounted provisions are estimated with the expected value, i.e. the expected value of the provisions in one year is equal to the current provisions. In the formulation introduced above, this means $E[C_{PY}]=1$. For the discounted provisions to achieve the expected technical result of zero, the additional assumption (18) is necessary.

If assumptions are made about the future interest for the liabilities, the same assumptions must also apply to the assets, with a corresponding effect on the expected financial result.

As an example, we shall look at a risk-free zero coupon bond that pays the value a in ten years. Its current value is

$$A(0)=\frac{a}{(1+r_{10}^{(0)})^{10}}$$

The value in one year will be

$$A(1)=\frac{a}{(1+R_9^{(1)})^9}.$$

If we calculate the expected value in order to receive the expected one-year performance, we obtain:

$$\frac{E[A(1)]}{A(0)}=\frac{E\left[\frac{1}{(1+R_9^{(1)})^9}\right]}{\frac{1}{(1+r_{10}^{(0)})^{10}}}=1+r_1^{(0)},$$

where precisely the abovementioned assumption about future interest has been made. The result is an expected performance of this asset in the amount of the one-year risk-free interest rate.

This is not market-consistent, since the asset with a holding period of one year is exposed to an interest rate risk, even though the payment will certainly be made in ten years. For this reason, a risk-averse market will therefore require a higher expected performance than the performance calculated here.

This entails the conclusion that the interest assumption (18) is questionable in a risk-averse market. The standard model makes this assumption, however, since it leads to a simplification with respect to liabilities. Any other assumption about future interest leads to a settlement result on a discounted basis that is non-zero. Since interest assumptions apply both to liabilities and to assets, the expected performance in the standard model of a Confederation bond is equal to the one-year risk-free interest rate.

Deviating from assumption (18) in favour of a different assumption about the expected value of the future interest rates is permissible, if this assumption applies both to assets and to liabilities.

If it is assumed that a long-term Confederation bond will achieve higher performance than the one-year risk-free interest rate, this leads to an expected loss on discounted liabilities. In a portfolio with an identical payment structure of assets and liabilities, the two effects cancel each other out exactly.

4.4.5. The expected result

The right side of formula (17b) consists of four terms. The second and the third term, both with the expected value of zero, describe the market and insurance risk. The expected change of the risk-bearing capital is captured in the first and fourth term. They concern the expected technical result and the expected financial result.

It is very important to note the comment in the preceding section 4.4.4.5 that the same assumptions about the future interest-rate curve must be made for assets and for liabilities.

4.4.5.1. Expected technical result

The expected technical result consists of the expected earned premiums, minus the discounted expected value of the occurring new claims and the expected costs (17b). If, as is the case in the standard model, assumption (18) is made, the provisions for PY claims do not contribute to the expected technical result.

Accident insurance (UVG) annuities are the exception. As described in section 4.4.4.4, UVG annuities result in an expected technical loss in the amount of $(E[\phi_{10/10}] - r_1^{(0)}) \cdot L_0$.

4.4.5.2. Expected financial result

The expected financial result is equal to the performance of the assets minus the one-year risk-free interest rate. The reason for deducting the risk-free interest rate is found in the definition of the target capital (2b) and (4). This is consistent with the fact that holding a portfolio consisting exactly of a one-year Confederation bond results in a target capital of zero.

If, as in the standard model, assumption (18) is made, then the performance of the risk-free bonds is exactly equal to the one-year risk-free interest rate. Accordingly, they do not contribute to a performance above the one-year risk-free interest rate.

4.4.6. Determination of the distribution for the technical result arising from CY claims

In the following, we will provide a description of how the claims expense S_{CY} is modelled in the standard model. It should be noted that the difference between the variable and its expected value $E[S_{CY}]$ must be used for the aggregation in formula (17b).

To model the annual claims expense S_{CY}, a distinction is made between minor claims (normal claims) and major claims. The reason is that no reasonable probability distribution exists that describes both minor and major claims. For purposes of the SST, the boundary between minor and major claims (major claim threshold β) can be chosen to be CHF 1 million or CHF 5 million.
Major claims encompass both individual major claims (by line of business) and cumulated claims, for instance caused by natural phenomena such as hail or floods. Cumulated claims may extend across lines of business. For instance, a hailstorm affects property insurance, but especially also comprehensive motor vehicle insurance.

Accordingly, we are looking for the distribution of the total claims expense as a sum of normal claims and major claims:

$$S_{CY} = S_{CY}^{NS} + S_{CY}^{GS} \,.\, S^{NC}{}_{CY} + S^{MC}{}_{CY} \tag{20}$$

The two distributions for the claims expense for normal and major claims must be found. The SST assumes that the major claims are independent of the normal claims. This entails that the aggregation of the two claims types into S_{CY} results from folding the two distributions.

4.4.7. CY claims: Distribution of normal claims

With one exception, the usual classification of lines of business as presented in appendix 8.4.1 is used for small CY claims. The exception concerns the normal claims in the natural hazard pool, since such claims are a component of the separate NHP modelling (see section 4.4.9).

The annual claims expense for normal claims is composed of the individual claims of the lines of business. The SST does not make any explicit assumption about the distribution of the individual claims; instead, the annual claims expenses are only represented with their expected value and variance.

The following sections describe how the expected value and the variance can be calculated for all normal claims (across all lines of business):

4.4.7.1. Expected value

The expected value for the entire normal claims expense S_{CY}^{NS} S^{NC}_{CY} can be calculated as the sum of the expected value per line of business i:

$$E(S_{CY}^{NS}) = \sum_{i=1}^{12} E(S_{CY,i}^{NS}) \,.\; S^{NC}_{CY} \tag{21}$$

The expected claims expenses $E(S_{CY,i}^{NS})$ per line of business can, for instance, be estimated as the product of the expected claims rate and the expected value of the earned premium:

$$E(S_{CY,i}^{NS}) = LR_i \cdot P_i,\ (i = 1,2,\ldots,12)\ \ S^{NC}_{CY} \tag{22}$$

4.4.7.2. Variance

In this section, we will explain the variance of the total normal claims expense based on the variances and covariances of the normal claims expenses of the lines of business. Then we will explain how the variances can be determined by line of business.

The variance of the total normal claims expense is calculated as the sum of the variances and covariances of all lines of business:

$$\begin{aligned} VAR(S_{CY}^{NS}) &= \sum_{i=1}^{12} VAR(S_{CY,i}^{NS}) + \sum_{i,j=1\, und\, i\neq j}^{12} Cov(S_{CY,i}^{NS}, S_{CY,j}^{NS}) \\ &= \sum_{i=1}^{12} (VK_i \cdot E(S_{CY,i}^{NS}))^2 + \sum_{i,j=1\, und\, i\neq j}^{12} \rho_{i,j} \cdot (VK_i \cdot E(S_{CY,i}^{NS})) \cdot (VK_j \cdot E(S_{CY,j}^{NS})) \end{aligned} \tag{23}$$

S^{NS}_{CY}->S^{NC}_{CY}

with VK_i as the variation coefficient of line of business i, defined by:

$$VK_i = \frac{\sigma(S_{CY,i}^{NS})}{E(S_{CY,i}^{NS})},\ \ S^{NS}_{CY}\text{->}S^{NC}_{CY} \tag{24}$$

with $\sigma(S_{CY,i}^{NS})$ NS->NC as the standard deviation of the normal claims expense $S_{CY,i}^{NS}$ $S^{NC}_{CY,i}$ in line of business number i and $\rho_{i,j}$ as the correlation coefficient for lines of business i and j. The correlation matrix valid in the standard model is given in appendix 8.4.2.

4.4.7.3. Variation coefficients

The contributions to the variance of the annual claims expense come from two sources. First, statistical fluctuations of the number and magnitude of claims around the expected value occur. This contribution to the uncertainty is called random or process risk. Second, an uncertainty exists with respect to the parameter of the distribution, or in other words with respect to the expected value and the variance. These quantities are namely not known, but rather must be estimated on the basis of statistics and expert knowledge, which are associated with uncertainty. The associated risk is called parameter risk. Examples of parameter risk are: wrong inflation estimates for life insurance, wrong estimates of claims frequencies, external changes, etc. A more precise consideration is given by the expression

$$Var(S^i_{CY,i}) = Var(E[S^{NS}_{CY,i}|\Theta_i]) + E[Var(S^{NS}_{CY,i}|\Theta_i)] \quad S^{NS}{}_{CY}\text{->}S^{NC}{}_{CY} \tag{25}$$

for the total variance for the claims expense of line of business number i. The first term represents the parameter risk, i.e. the variability of the model parameters from one year to the other that is caused by external circumstances. The totality of these circumstances is characterized by the random variable (risk parameter) Θ. Θ can be viewed as the risk characteristic of a fixed claims year. It measures how precisely an actuary can estimate the expected expenditure, or what external influences cannot be buffered by the risk equalization in the collective. (For this risk, the size of the company plays no role, which means that it cannot be diversified away.)

The second addend is the random risk that consists in the uncertainty of the annual claims amount given the risk parameter Θ_i (i.e. given the expected value and the variance of the distribution).

Assuming that for a given Θ_i, the number of claims in line of business i has a Poisson distribution with Poisson parameter λ_i (=expected number of claims), then the variation coefficient of $S^{NS}_{CY,i}$ $S^{NC}{}_{CY,i}$ is (derivation see appendix 8.4.5)

$$VK_i{}^2 = \frac{Var(S^{NS}_{CY,i})}{(E[S^{NS}_{CY,i}])^2} = VK_{p,i}{}^2 + \frac{1}{\lambda_i}(VK^2(Y_{i,j})+1), \quad S^{NS}{}_{CY}\text{->}S^{NC}{}_{CY} \tag{26}$$

where $VK(Y_{i,j})$ denotes the variation coefficient of the individual claims amount in line of business i. The first addend $VK_{p,i}$ represents the contribution of the parameter risk, and the second addend represents the contribution of the random risk. The following sections will discuss the parameter risk and the random risk in more detail.

4.4.7.4. Parameter risk

The variation coefficient of the parameter risk ($VK_{p,i}$) of line of business i is composed of a parameter uncertainty with respect to the expected value of the individual claims amount ($E[Y_{i,j}]$) and a parameter uncertainty with respect to the expected value of the number of individual claims ($E[N_i]$). These uncertainties in the parameters arise from external circumstances that affect many, if not all, companies. For this reason, standard values for the variation coefficients of the parameter risk have been determined for each line of business on the basis of common statistics of the insurers and are made available here (appendix 8.4.3).

4.4.7.5. Random risk

The variability of the j-th individual claims amount $Y_{i,j}$ in line of business i is represented by the variation coefficient $VK(Y_{i,j})$. The contribution in the parentheses results from the variability of the number of claims (for a given Θ, this has a Poisson distribution with the expected value λ_i). For the non-life SST, standard values for the variation coefficients of the individual claims amounts are made available for each line of business.

Numeric values for $VK(Y_{i,j})$ are given in appendix 8.4.4.

If the standard values for $VK_{p,i}$ and $VK(Y_{i,j})$ are used, the calculation of the variances (and of the variation coefficients) using formula (26) has the useful quality that only the expected number of claims per line of business has to be determined.
Using the abovementioned aggregation across lines of business provides the variance of the normal claims distribution (see (23) and (26)).

4.4.8. CY claims: Distribution of major claims

Major claims encompass both individual major claims and cumulated claims:

- Individual claims with a large claims amount. Such claims may for instance arise in the property (e.g. fire in a factory building) and liability lines of business (e.g. product liability or motor vehicle liability). As a first approximation, the amount of an individual claim does not depend on the insuring company.

- Cumulated claims: a group of claims triggered by the same event (e.g. hail or storm). The individual claims are generally not major claims, but the total claims expense may be high due to the large number of individual claims. Although the individual claims are not greater than the normal claims, they cannot be represented in the normal claims model due to their mutual dependency (cumulated event).

The SST major claims model considers the following event types and lines of business:

Line of business (or type of event)	**Comments on modelling major events**
MVL	modelled as individual major claim
MVC	encompasses hail claims, modelled as market share of market-wide claims
Property without natural hazards	modelled as individual major claims; without natural hazards, since these are modelled separately
General liability	modelled as individual major claim
Health, collective	modelled as individual major claim
Health, individual	modelled as individual major claim
Aviation	no modelling of major claims, since the aviation pool is heavily reinsured
Transport	modelled as individual major claim
Finance and surety	modelled as individual major claim
Accident (UVG and non-UVG)	modelled as market share of market-wide cumulated claims (e.g. panic in a football stadium) (=unknown cumulated); cumulated claims that only affect an individual insurer are not modelled directly as major claims, but rather are taken into account with the help of a scenario (=known cumulated).
Natural hazard pool	modelled as share of the market-wide claims, according to participation in the natural hazard pool
Other natural hazards	modelled as share of business interruption or another appropriate quantity relative to the market-wide claims. The market-wide claims are modelled as < co-monotone quantity relative to major claims in the natural hazard pool.

Major claims modelling by line of business/type of event

Frequently, insurance undertakings participate in a cumulated claim according to their market share in the line of business in question, so that it makes sense to determine common parameters.

The major claims are modelled separately for each line of business of type of event i with a compound Poisson distribution:

$$S_{CY,i}^{GS} = \sum_{j=1}^{N_i^{GS}} Y_{i,j}^{GS} \text{ . GS->MC} \tag{27}$$

Here, the number of claims N_i^{GS} GS->MC of line of business i has a Poisson distribution with the expected value λ_i^{GS} GS->MC. It is assumed that the individual gross claims $Y_{i,j}^{GS}$ GS->MC are distributed independently of each other and identically within the line of business or type i. A Pareto distribution is used for each type i:

$$F_{Y_{i,j}^{GS}}(y) = P(Y_{i,j}^{GS} \le y) = \begin{cases} 0 & y < \beta, \\ 1 - \left(\frac{y}{\beta}\right)^{-\alpha_i} & y \ge \beta. \end{cases} \text{ GS->MC} \tag{28}$$

β is the smallest claim considered in the major claim model, which is why β is often called the major claim "threshold". Another name is "observation point of the Pareto distribution". The standard values of the parameters in the SST are designed so that β is either CHF 1 or 5 million. One of these values must be chosen by each insurer. This choice can be made individually for each line of business, but the notation " β " in this document does not take this account.

Pareto distributions have the quality that they assign greater weight to high claims than many other distributions. The strength of the weighting is determined by the Pareto parameters α_i. The smaller the value of α_i, the more weight the higher major claims have. The standard values for the Pareto parameters are::

Line of business	**for β =CHF 1 million**	**for β =CHF 5 million**
MVL	2.50	2.80
MVC-hail	1.85	1.85
Property	1.40	1.50
Liability	1.80	2.00
UVG incl. UVGZ	2.00	2.00
Health, collective	3.00	3.00
Health, individual	3.00	3.00
Transport	1.50	1.50
Finance and surety	0.75	0.75
Others	1.50	1.50

Parameters α_i of the Pareto distribution

By using the Pareto distribution for modelling the individual claims amounts, arbitrarily high claims amounts are possible in the model. In reality, however, claims in certain lines of business cannot be arbitrarily high, e.g. due to contractually agreed maximum insurance amounts. It therefore makes sense to truncate the Pareto distribution at a certain value. For this purpose, standard truncation points (limits) are determined for some lines of business.

These guidelines are listed in the following table. They are not binding, but deviations must be explained.

Line of business	Truncation point
MVL	Unlimited
MVC	Market share × CHF 1.5 billion
Property	Individual estimate of the largest possible claim for each insurer
Liability	Individual estimate of the largest possible claim for each insurer
UVG incl. UVGZ	Unlimited
Accident without UVG	CHF 50 million
Health, collective	Individual estimate of the largest possible claim for each insurer
Health, individual	Individual estimate of the largest possible claim for each insurer
Transport	2 × largest possible insurance amount
Finance and surety	Individual estimate of the largest possible claim for each insurer
Others	Individual estimate of the largest possible claim for each insurer
Natural hazards in the natural hazard pool	At the contractually agreed CHF 500 million per event for the market-wide claims.
Natural hazards not in the natural hazard pool	Market share × CHF 1 billion

Truncation points of the Pareto distribution. Natural hazards are discussed in section 4.4.9 and only mentioned here for the sake of completeness.

The following sections will discuss hail claims, cumulated accident claims, and natural hazards in more detail, since these claims are not specific to a particular insurance company, but rather affect all insurers with the same cumulated claims. For this reason, the market-wide claims are modelled and scaled down to the claim of an individual insurance company by multiplying with the market share of the insurer.

4.4.8.1. Modelling of cumulated claims due to hail events

The modelling of cumulated hail claims mainly concerns comprehensive motor vehicle insurance. As in other lines of business, major hail claims are also represented with a Poisson and Pareto distribution.

First, the threshold for the market-wide major hail claims is fixed at $\beta_{Hagel}^{(Markt;0)}$ Markt->market, Hagel->hail= CHF 45 million. An estimate based on extensive claims statistics generates the following parameters: The expected value of the number of claims > CHF 45 million is $\lambda_{Hagel}^{(0)}$ Hagel->hail =0.9. The Pareto parameter is α_{hail}=1.85.

The modelling also requires an individual major claims threshold per company, however. This threshold depends on the company's chosen individual major claims threshold β and the market share of the hail claims m_{hail} (this can be equal to the market share with respect to comprehensive motor vehicle insurance.)

Example: If the company's own major claims threshold is β = CHF 1 million and the market share is m=10%, then the market-wide claims above

$$\beta_{Hagel}^{(Markt)} = \frac{\beta}{m_{Hagel}} = \frac{1}{0.1} MCHF = 10 MCHF \quad \text{Markt->market, Hagel->hail} \tag{29}$$

must be included in the calculation instead of $\beta_{hail}^{(0)}$.

The expected number for the individual threshold is generated by the Pareto distribution as

$$\lambda_{Hagel} = \lambda_{Hagel}^{(0)} \cdot \left(\frac{\beta_{Hagel}^{(Markt)}}{\beta_{Hagel}^{(Markt;0)}} \right)^{-\alpha} . \quad \text{Markt->market, Hagel->hail} \tag{30}$$

In the following example, the expected value to be used is

$$\lambda_{Hagel} = 0.9 \cdot \left(\frac{10}{45} \right)^{-1.85} = 14.5 . \quad \text{Hagel->hail} \tag{31}$$

Note: Actually, it is incorrect to extrapolate backwards from CHF 45 million claims to CHF 10 million using the Pareto distribution, since the Pareto distribution no longer applies in this low area, i.e. it would be incorrect to assume that the expected number of hail claims over CHF 10 million would be 14.5. In reality, it is lower.
However, this error can be ignored in the major claims model, since only the behaviour in the tail (i.e. in the case of higher claims) of the distribution is relevant. With respect to the behaviour in the tail of the distribution, the approximation is good.

The Pareto distribution for the event claims amount of the market-wide hail claims can be truncated at CHF 1.5 billion. Accordingly, it has the following form in the case of the example above:

$$F_{Y_{Hagel,j}^{GS}}(y) = \begin{cases} 0 & y < 10MCHF, \\ 1 - \left(\frac{y}{10MCHF} \right)^{-1.85} & 10MCHF \le y \le 1500MCHF, \\ 1 & 1500MCHF < y. \end{cases} \quad FY^{MC}_{hail,\,j} \tag{32}$$

4.4.8.2. Cumulated events in accident insurance (unknown cumulation)

Like hail claims, the cumulated claims in accident insurance are modelled as the individual market share m_{Unfall} $m_{accident}$ in a market-wide cumulated claim.

The market-wide claim distribution is a compound Poisson distribution with threshold $\beta_{Unfall}^{(Markt;0)}$ Markt->market, Unfall->accident= CHF 20 million. The probability that a cumulated claim greater than or equal to β_{Unfall} $\beta_{accident}$ occurs for private insurers (i.e. without SUVA) in a given year has been estimated as $\lambda_{Unfall}^{(0)}$ Unfall->accident = 0.1. The Pareto parameter is $\alpha_{accident}$=2.

As for the treatment of cumulated hail claims, an adjustment of the smallest observed market-wide claim $\beta_{Unfall}^{(Markt)}$ Markt->market, Unfall->accident and the expected frequency λ_{Unfall} Unfall->accident is necessary so that they are consistent with the company's own major claims threshold and the market share.

$$\beta_{Unfall}^{(Markt)} = \frac{\beta}{m_{Unfall}},$$ Markt->market, Unfall->accident (33)

$$\lambda_{Unfall} = \lambda_{Unfall}^{(0)} \cdot \left(\frac{\beta_{Unfall}^{(Markt)}}{\beta_{Unfall}^{(Markt;0)}} \right)^{-\alpha_{Unfall}}.$$ Markt->market, Unfall->accident (34)

It is suggested to measure the market share using the earned premium before reinsurance.

4.4.8.3. Aggregation of the major claims distribution

This section explains how the abovementioned major claims distributions (compound Poisson distributions by line of business) can be aggregated in a simple way. First, this relies on the fact that the sum of independent variables with a compound Poisson distribution again has a compound Poisson distribution. Second, a compound Poisson distribution can be numerically derived with the Panjer algorithm in a simple manner.

As a first step, we reiterate that the major claims distribution by line of business or type of event is given by the stochastic sum of individual gross claims $Y_{i,j}^{GS,B}$ $Y^{MC,G}$

$$S_{CY,i}^{GS,B} = \sum_{j=1}^{N_i^{GS}} Y_{i,j}^{GS,B}.$$ $S^{MC,G}_{CY,i}$ … N^{MC}_{i}…$Y^{MC,G}_{i,j}$ (35)

The index i stands for one of the lines of business with individual major claims and for the events "cumulated hail claim" and "cumulated accident claim". The claims of the natural hazard pool are not yet included in the aggregation at this point.

The distribution of the numbers of claims by line of business/type of event i has a Poisson distribution:

$$N_i^{GS} \sim Poisson(\lambda_i^{GS}).$$ GS -> MC (36)

The gross claims amount distribution by line of business/type of event i is:

$$Y_{i,j}^{GS,B} \sim Pareto(\alpha_i, \beta),$$ $Y^{MC,G}_{i,j...}$ (37)

or

$$Y_{i,j}^{GS,B} \sim Pareto(\alpha_i, \beta_i),$$ $Y^{MC,G}_{i,j...}$ (38)

if the major claims threshold is chosen individually by line of business.
The distribution of the net claims amount per claim is derived by applying the XL coverages to the Pareto distribution. The result is formally described here with

$$Y_{i,j}^{GS,Netto} \sim F_{Y_{i,j}^{GS,N}}.$$ $Y^{MC,Net}_{i,j}$~F $Y^{MC,N}_{i,j}$ (39)

The sum of the annual claims expenses across lines of business/types of event (net and gross)

$$S_{CY}^{GS} = \sum_i S_{CY,i}^{GS} = \sum_i \sum_{j=1}^{N_i^{GS}} Y_{i,j}^{GS}$$ GS -> MC (40)

again has a compound Poisson distribution (here without derivation), i.e. it can be written as

$$S_{CY}^{GS} = \sum_{k=1}^{N^{GS}} Y_k \text{ , GS -> MC} \tag{41}$$

where

$$N^{GS} \sim Poisson\left(\sum_i \lambda_i^{GS}\right) \text{ GS -> MC} \tag{42}$$

applies to the distribution of numbers of claims, while the distribution of the individual claims Y_k is constructed as follows as a weighted average of the distribution function of the individual claims of the individual lines of business/types of business:

$$F_Y(y) = \frac{1}{\sum_i \lambda_i} \cdot \sum_i \left(\lambda_i \cdot F_{Y_{CY,i}^{GS}}(y)\right). \text{ GS -> MC} \tag{43}$$

This total major claims distribution S can now be calculated with the fast Fourier transformation or preferably with the Panjer algorithm.

The aggregation of major claims described here is not yet able to include the distribution of the natural hazard claims. The reason is that the natural hazard pool has a stop-loss coverage. This entails that the distribution of the net annual claims from natural hazards does not have a compound Poisson distribution, and accordingly that the distribution cannot be aggregated with the distributions of the other major claims using the method described above. Instead, it must be folded into S_{CY}^{GS} $S^{MC}{}_{CY}$ after the fact.

4.4.9. Modelling of natural hazard claims

The reason why natural hazard claims are modelled separately from other major and normal claims is the existence of stop-loss coverage for the natural hazard pool (NHP). If this coverage were not taken into account, natural hazards could be treated like other major claims and aggregated with them.

Natural hazard claims encompass the property damage that is caused by natural hazard events. On the one hand, these include the major and normal claims affecting the NHP. These concern damage to buildings (in the GUSTAVO cantons[E]) and damage to chattels/building contents.
Major natural hazard events also give rise to claims in other lines of business, such as business interruption (BI). This second type is called "other natural hazards" here. Since the other natural hazards are closely linked to the claims in the NHP, they are modelled together with NHP claims.

The following two sections will describe the modelling of the NHP, followed by other natural hazards.

4.4.9.1. Modelling of the natural hazard pool

Modelling of the natural hazard pool relies on the following assumptions and simplifications:

- The insurers for buildings and chattels are members of the natural hazard pool. The insurers participate individually in the NHP claims depending on their market share of fire insurance.
- The NHP effects a 100% redistribution of claims, not merely 85%.
- Per event, the insurance industry does not pay more than CHF 500 million (CHF 250 million event limit each for buildings and content. It is foreseeable that this limit will be raised, but the amendment has not yet entered into force). The insurers do have the option to pay a larger amount than this limit in the event of major claims for reasons of reputation, which in fact happened for the claims in the summer of 2005. However, the supervisory authority does not

[E] Geneva, Uri, Schwyz, Ticino, Appenzell Innerroden, Valais, and Obwalden want to require any target capital for such cases, since there is no legal duty to make such payments.

- The pool is covered by an annual stop loss.
- The NHP claims are divided into major and normal claims, where the major claims are defined as the NHP event claims with a claims amount greater than or equal to CHF 50 million.

All natural hazard insurers are affected by all NHP claims to a certain percentage. For this reason, it makes sense in a first step to model the market-wide claims. In a second step, the resulting distribution will be divided according to market share.

Major claims

The model for the annual sum of the major claims for the NHP is

$$S_{ESP}^{GS} = \sum_{j}^{N} \min(500, Y_j) \quad S^{MC}{}_{NHP...} \tag{44}$$

consisting of a compound Poisson distribution, where the market-wide event claims amounts Y_j follow a generalized Pareto distribution, but are truncated at CHF 500 million.

The cumulative distribution function of the generalized Pareto distribution has the form

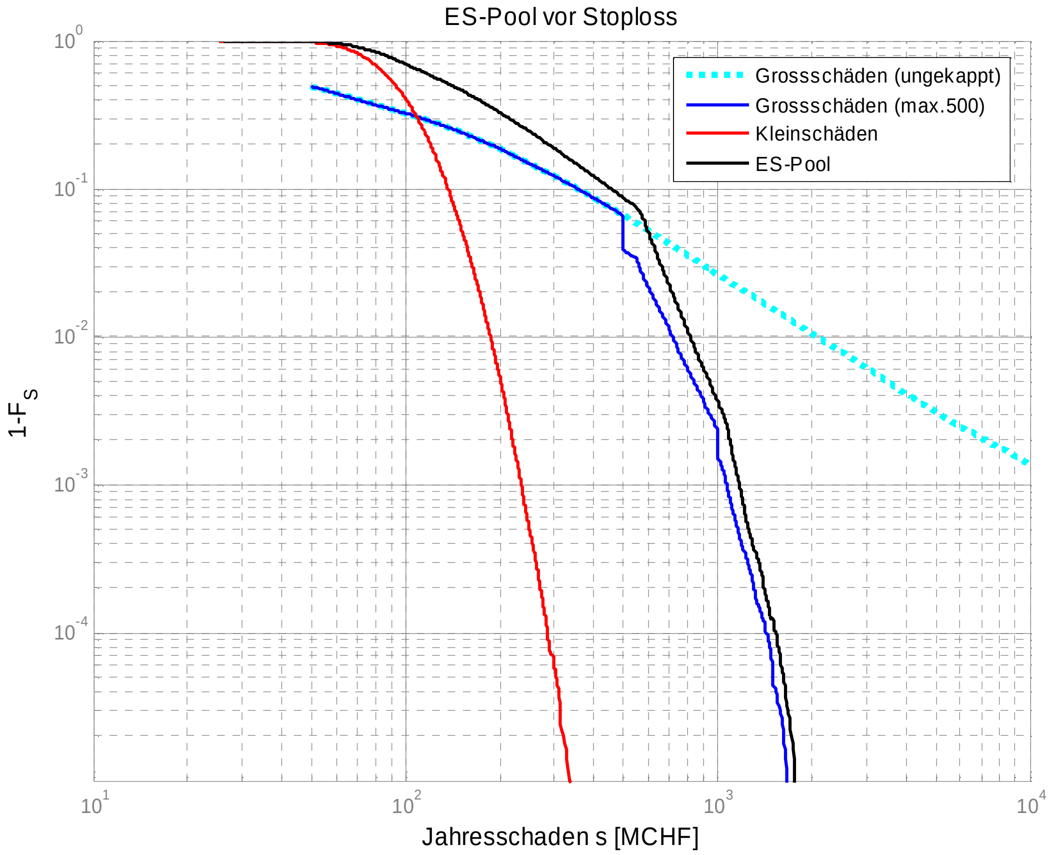
$$F(x) = \begin{cases} 1 - \left(\dfrac{x_0 + b}{x + b} \right)^{\alpha} & x \geq x_0 \\ 0 & x < x_0 \end{cases}$$

Like the Pareto distribution, $1 - F(x)$ for $x >> x_0 + b$ behaves like $\sim x^{-\alpha}$.

The evaluation of the data from the natural hazard pool conducted by the Swiss Insurance Association (SIA) using $x_0 = 50$ million CHF generated the parameters $\alpha = 1.2499$ and $b = 18.7761$ million CHF for the event claims amount and an annual expected number of major claims events greater than or equal to 50 million CHF of $E[N] = \lambda = 15/22 = 0.68687$ (in 22 years, 15 events with inflation-adjusted claims amounts greater than CHF 50 million).

Normal claims

In addition to the major claims, there is the annual sum S_{ESP}^{KS} KS->NC, ESP->NHP of the normal claims of the NHP. For these variables, the SIA evaluation assumes a lognormal distribution with the moments $E[S_{ESP}^{KS}] = 97.48$ KS->NC, ESP->NHP million CHF and $\sqrt{Var(S_{ESP}^{KS})} = 0.3072 \cdot E[S_{ESP}^{KS}]$ KS->NC, ESP->NHP. For purposes of estimating risk, it is even permissible to set the variance of S_{ESP}^{KS} KS->NC, ESP->NHP to zero, since the NHP risk is dominated by the major claims (see also Figure 6).



NH pool before stop loss

Major claims (uncapped)
Major claims (max. 500)
Minor claims
NH pool

Annual claims S [MCHF]

Figure 5: Probabilities of exceeding the annual claims sum of the natural hazard pool before the stop loss. The figure represents the normal claims (red), the major claims (blue), taking into account the event limit of CHF 500 million, and the sum of both (black line). The light-blue dotted line shows the distribution of the major claims without an event limit. The expected shortfall for the three first categories is 208 MCHF, 880 MCHF, and 982 MCHF.

Sum of major and normal claims, application of stop loss

The annual stop loss (750 xs 450 million CHF) of the NHP influences the sum of the major and normal claims. Formally, we can write the annual claims remaining in the NHP as

$$S_{ESP} = \mathbf{SL}_{750xs450}\{S_{ESP}^{KS} + \sum_{i=1}^{N} \min(500, Y_i)\}\ , \text{ KS->NC, ESP->NHP}$$

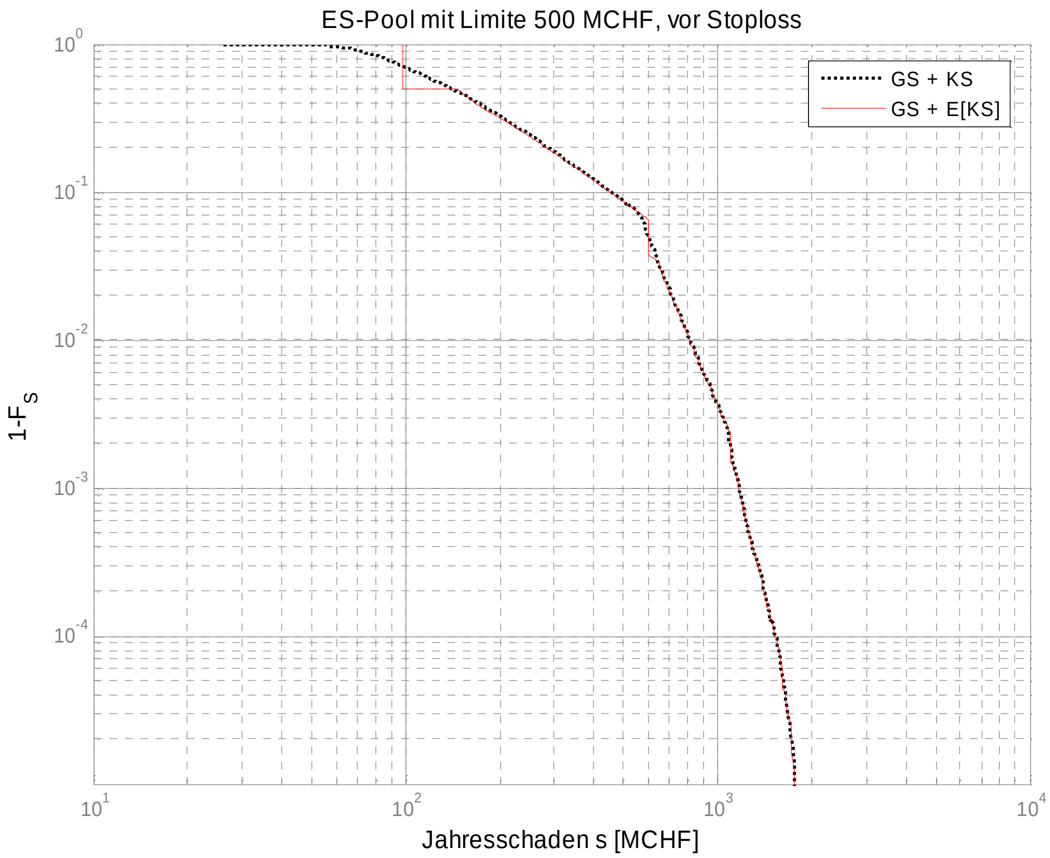
where the stop loss is defined as

$$\mathbf{SL}_{750xs450}\{x\} := \min[x, \max(x - 750\ \text{MCHF}, 450\ \text{MCHF})]\,.$$

In the standard model, it can be assumed that the normal claims sum is characterized by its expected value sufficiently precisely. This results in

$$S_{ESP} = E[S_{ESP}^{KS}] + \mathbf{SL}_{750xs350}\{\sum_{i=1}^{N} \min(500, Y_i)\}\ , \text{ KS->NC, ESP->NHP}$$

since the expected value of the normal claims sum is quite precisely CHF 100 million.



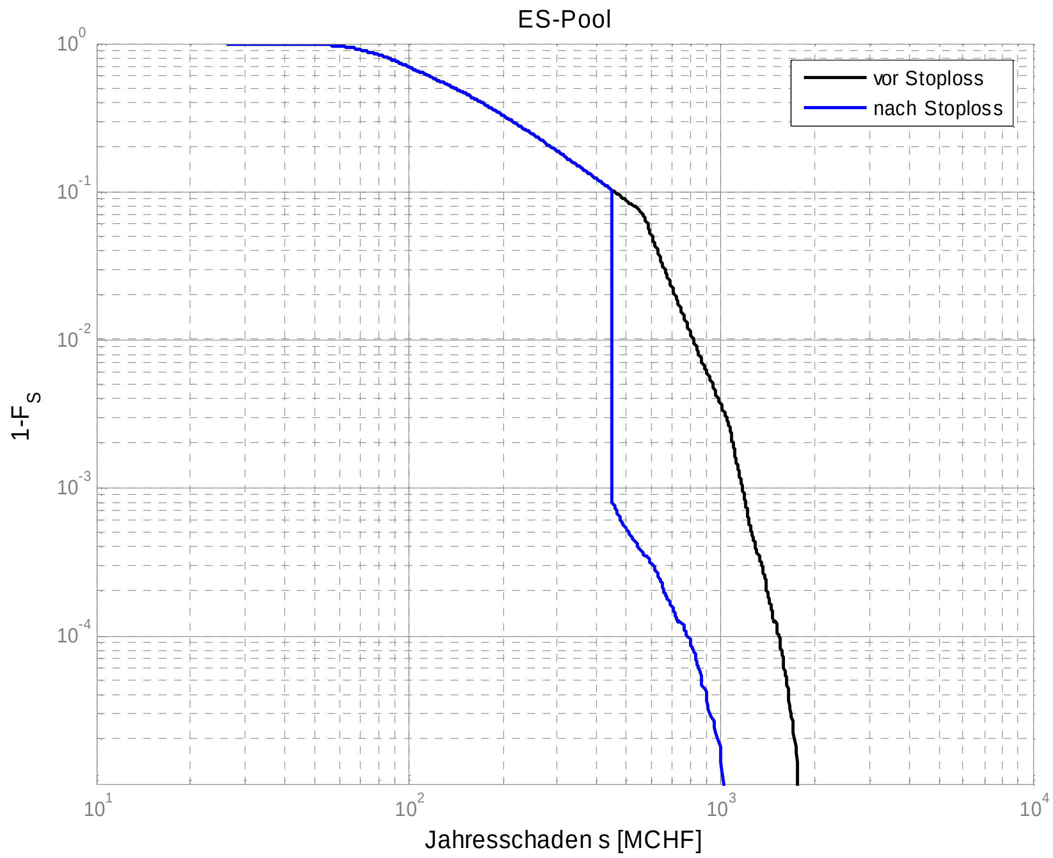
NH pool with limit 500 MCHF, before stop loss

MC + NC
MC + E[NC]

Annual claims S [MCHF]

Figure 6: Probabilities of exceeding the annual claims sum of the natural hazard pool before

the stop loss. The figure represents the effect of the simplifying assumption that the normal claims are not modelled with a lognormal distribution (black, dotted line), but rather with the deterministic value of the expected value (red line).



NH pool

before stop loss
after stop loss

Annual claims S [MCHF]

Figure 7: Probabilities of exceeding the annual claims sum of the natural hazard pool before and after the stop loss (750 xs 450). The expected shortfall is CHF 982 million before the stop loss and CHF 460 million after the stop loss.

4.4.9.2. Modelling of "other natural hazards"

Experience shows that larger natural hazard events not only entail damage to buildings and chattels, but also trigger other insured losses. The most important has been business interruption. The flood events of August 2005 have shown that these additional claims amount to approximately

- 20% of the NHP claims for property
- 10% of the NHP claims for MVC.

The standard model of the SST assumes that the other property claims fully correlate with the NHP claims. However, the standard model only models the other natural hazard claims in the property line of business. Possible MVC natural hazard claims are already covered by the independent distribution for hail claims.

Where, like above, Y_j denotes a major NHP claim, an additional claim of $0.2 \times Y_j$ therefore arises. For these claims as well, it makes sense to introduce an upper threshold. This threshold is fixed at CHF 1000 million. The annual sum is

$$S^{GS}_{\ddot{u}br.Elementar} = \sum_{j}^{N} \min(1000\,\mathrm{MCHF}; 0.2 \cdot Y_j) \quad S^{MC}_{\text{other NH...}}$$

4.4.9.3. Modelling the NHP + "other natural hazards"

The sum of the NHP claims and the other natural hazard claims for the insurance market generates the following expression:

$$S^{Markt}_{Elementar} = E[S^{KS}_{ESP}] + \mathbf{SL}_{750xs350}\{\sum_{i=1}^{N} \min(500, Y_i)\} + \sum_{i=1}^{N} \min(1000; 0.2 \cdot Y_i)$$

$S^{market}_{NH} = E[S^{NC}_{NHP}] + \ldots$

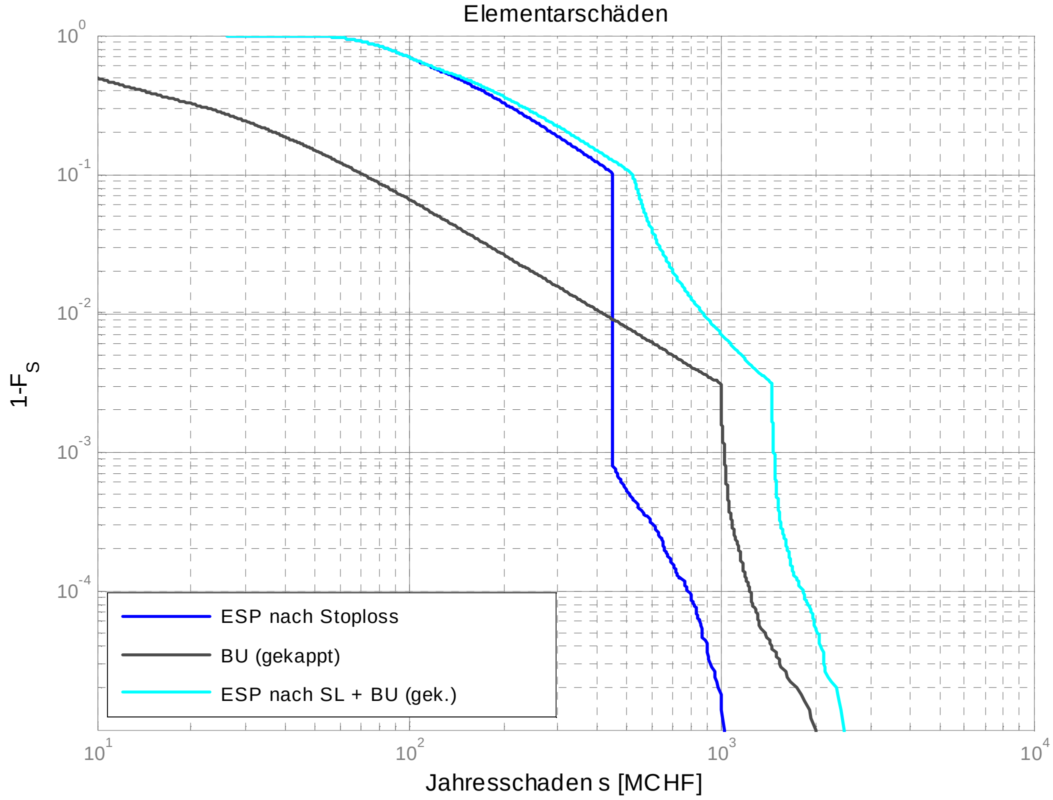
The individual insurer participates in this to a lesser or greater extent, depending on how large its business volumes are. With the market share m_{ESP} m_{NHP} in fire insurance, which is considered relevant to the NHP, and the market share m_{BU} m_{BI} in business interruption insurance, the following equation results for the individual insurer:

$$S^{individuel}_{Elementar} = m_{ESP} \cdot \left\{ E[S^{KS}_{ESP}] + \mathbf{SL}_{750xs350}\{\sum_{i=1}^{N} \min(500, Y_i)\} \right\} + m_{BU} \cdot \sum_{i=1}^{N} \min(1000; 0.2 \cdot Y_i).$$

$S^{individual}_{NH} = m_{NHP} \ldots E[S^{NC}_{NHP}] + \ldots + m_{BI} \ldots$

The distribution of this quantity must be calculated and finally aggregated with the results of the minor and major claims of the other lines of business.

The figure shows the distributions of NHP after the stop loss and the BI claims capped at CHF 1000 million.



Natural hazards

NHP after stop loss
BI (capped)
NHP after SL + BI (capped)

Annual claims S [MCHF]

Figure 8: The probabilities of exceeding the annual claims sum of the natural hazard pool after the stop loss, the BI claims (capped at CHF 1000 million per event), and the sum of both. The expected shortfall is CHF 461, 744, and 1203 million.

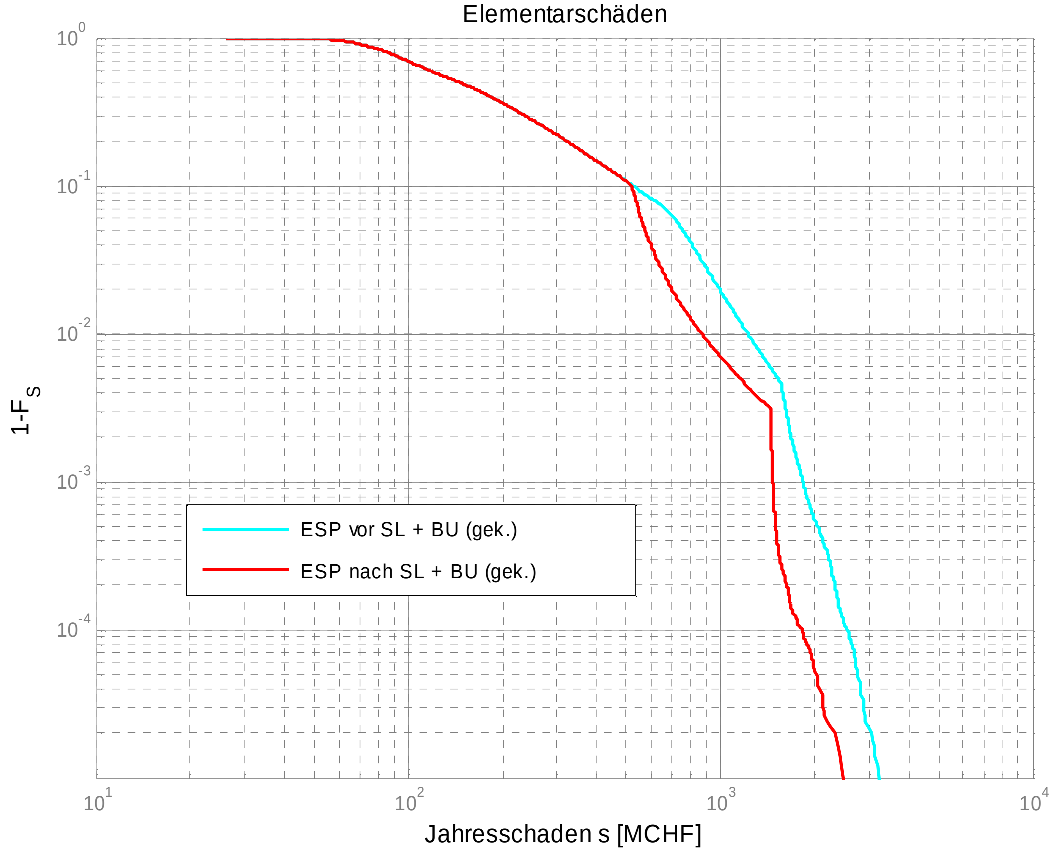
Possible simplification by omitting the stop loss

If consideration of the stop loss for the NHP is omitted, the calculation and the aggregation become simpler. The expression above can first be simplified as

$$S_{Elementar}^{individuel} = m_{ESP} E[S_{ESP}^{KS}] + \sum_{i=1}^{N} \left\{ m_{ESP} \min(500; Y_i) + m_{BU} \cdot \min(1000; 0.2 \cdot Y_i) \right\}$$

$S^{individual}{}_{NH} = m_{NHP} \ldots E[S^{NC}{}_{NHP}] + \ldots m_{NHP} \ldots + m_{BI} \ldots$

The distribution of the quantity in the curly brackets is easy to express numerically. The rest of the formula is then in a form that can be treated with the Panjer algorithm, exactly like the other major claims in the other lines of business.



Natural hazards

NHP before SL + BI (capped)
NHP after SL + BI (capped)

Annual claims S [MCHF]

Figure 9: Probability of exceeding the annual claims sum of the natural hazard pool and the BI claims (capped at CHF 1000 million). Red curve: NHP after stop loss, ES = CHF 1203 million; blue curve: NHP without stop loss, ES = CHF 1547 million. The omission of the stop loss leads to a conservative result, but the modelling is simpler, since the Panjer algorithm can be applied.

4.4.10. Determination of the distribution for the technical result from the PY

The risk of the provisions consists in the uncertainty of the settlement result.

The standard model assumes that the quantity $C_{PY} \cdot R_{PY}^{(0)}$ has a lognormal distribution, with a certain variance and the expected value $R_{PY}^{(0)}$. This implies that we assume best estimate provisions, $E[C_{PY}]=1$. This section discusses how the variance should be estimated.

As in the case of new claims, a distinction is made between random risk and parameter risk. The random risk consists of randomness that arises from inaccurate estimates of the individual claims. It is determined according to individual companies by estimating the variances

$$Var_Z(C_{PY,i} \cdot R_{PY,i}^{(0)}) \qquad (i = 1,...,12)$$

of the provisions of the 13 lines of business, using the time series of the historical settlement results. It is vital to determine the settlement results on the basis of best estimate provisions.

The parameter risk of the provisions arises when the estimates of parameters are uncertain that affect all provisions of a line of business at the same time, or if the level of the total claims provisions was chosen incorrectly.

The SST Working Group has not yet developed an ultimate recipe for determining the parameter risk. For this reason, FOPI currently predetermines the variation coefficients of the provisions with respect to parameter risk (see appendix 8.4.6). These figures rely on the average values for parameter uncertainties in large provision portfolios.
The variance of $C_{PY} \cdot R_{PY}^{(0)}$ with respect to the parameter risk can be calculated easily using the predetermined variation coefficients of the 13 lines of business as

$$Var_P(C_{PY,i} \cdot R_{PY,i}^{(0)}) = \left(R_{PY,i}^{(0)} \cdot Vko_P(C_{PY,i})\right)^2 \qquad (i = 1,...,13)$$

Parameter risk and random risk are aggregated by line of business by adding the variances.

$$Var(C_{PY,i} \cdot R_{PY,i}^{(0)}) = Var_P(C_{PY,i} \cdot R_{PY,i}^{(0)}) + Var_Z(C_{PY,i} \cdot R_{PY,i}^{(0)}) \qquad (i = 1,...,13)$$

In the 2005 test run, the aggregation of the risks across the lines of business is accomplished by adding the variances across lines of business (without covariances, which implies non-correlation between the lines of business). The assumption of non-correlation may be deviated from in the coming years.

$$Var(C_{PY} \cdot R_{PY}^{(0)}) = \sum_i^{12} Var(C_{PY,i} \cdot R_{PY,i}^{(0)})$$

This discussion entails that the expression $(1 - C_{PY}) \cdot R_{PY}^{(0)}$ has the variance $Var(C_{PY} \cdot R_{PY}^{(0)})$ and the expected value of zero.

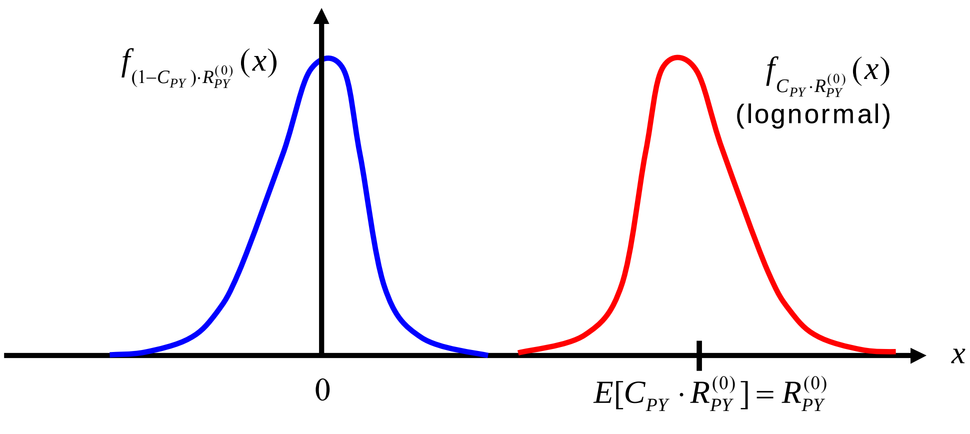


Figure: Schematic representation of the density distribution of $C_{PY} \cdot R_{PY}^{(0)}$ (red curve) and $(1 - C_{PY}) \cdot R_{PY}^{(0)}$ (blue curve).

4.4.10.1. Comments on the provision risks in accident insurance

Accident insurance is divided into:

- Compulsory accident insurance (UVG). This is concluded by employers for the collective of the employees.
- Supplements to compulsory accident insurance (UVG-Z). In most cases, these cover the same collective as the UVG insurance and offer coverages that go beyond the UVG coverage.
- Individual accident insurance (IAI): Accident insurance for individuals.

Accident insurance distinguishes between

- Short-term benefits: Benefits for therapies, daily allowances, prostheses, rehabilitation, etc.
- Long-term benefits: Annuity payments in case of inability to work, consisting of a basic annuity and the cost-of-living adjustment.

Provision risks for cases without or before annuity payments

After an accident has occurred, the insurer sets aside a claims provision for settling the claim, where the possibility of recourse to any liability benefits is taken into account. The claims provisions encompass the short-term benefits and, as a rule, also the long-term benefits. As in other lines of business, the provisions are subject to the provision risk.

Provision risk for long-term benefits (annuities)

(See also section 4.4.4.4.)

If an accident results in an annuity, the insurer determines an "annuity coverage capital" for its settlement. Depending on the insurer, this is separated out of the claims provisions or retained as part of them.
Since the annuity is fixed at the time of the annuity payment, no uncertainty risk applies for the insurer, with one exception. The exception is that the annuity payments must regularly be adjusted according to a cost-of-living index. Since future inflation is not yet known, the insurer assumes a risk.

At this point, we will not yet discuss how the cost-of-living is determined for annuities. The financing of cost-of-living adjustments is secured through the financial return on the investment annuity coverage capital, and if this is insufficient, by levying additional contributions from current policyholders (contribution premiums from active lives). In this way, the insurer can pass the inflation risk on to the active lives. If the insurer no longer has active policyholders, however, it can no longer levy contribution premiums. To remove this risk from the insurer, the Cost-of-Living Fund for UVG annuities was created. The situations are not yet precisely defined in which an individual insurer may receive support from the Cost-of-Living Fund.

The working hypothesis in the 2006 SST test run is that the Cost-of-Living Fund assumes the inflation risk of the UVG annuities. This means that the SST test run does not consider risks for UVG annuities.

4.4.11. Aggregation of technical risks

> Preliminary remark: In the standard model, the aggregation of provision risks and normal claims is performed either by folding the two distributions or, as a shortcut, using the simplifying assumption that the sum has a lognormal distribution.

The preceding sections explained how a distribution of the CY claims expense S_{CY} and of the settlement result $(1-C_{PY})\cdot R_{PY}^{(0)}$ can be reached.

However, we need a total distribution of the technical risk in accordance with formula (19):

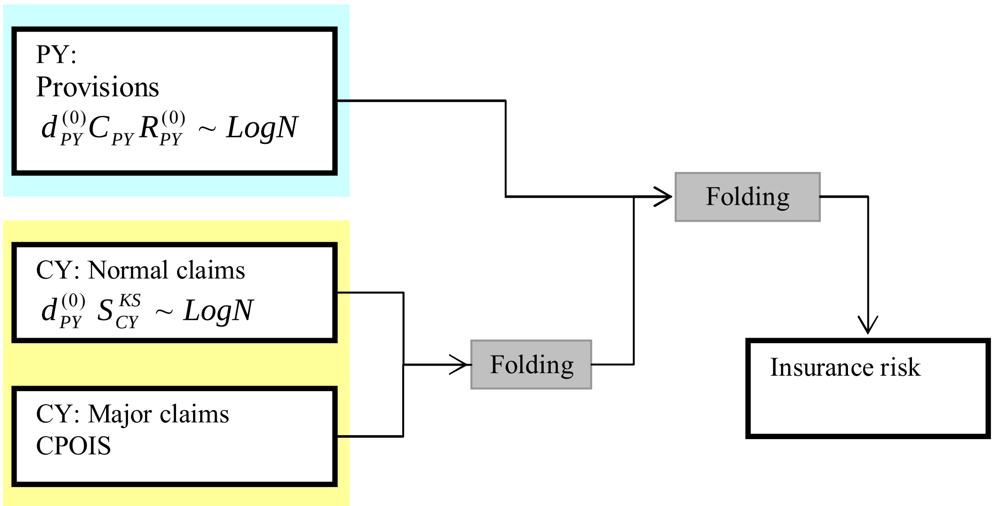
$$\left(P-K-d_{CY}^{(0)}E[S_{CY}]\right)-d_{CY}^{(0)}\left(S_{CY}-E[S_{CY}]\right)-d_{PY}^{(0)}\left(C_{PY}-1\right)R_{PY}^{(0)}$$

When using this formula, centred around the expected value $E[S_{CY}]$, the following steps must therefore be performed:

- Centring of S_{CY} around $E[S_{CY}]$. This gives us a distribution of $(S_{CY} - E[S_{CY}])$.
- Discounting of $(S_{CY} - E[S_{CY}])$ with $d_{CY}^{(0)}$.
- Discounting of the settlement result $(1 - C_{PY}) \cdot R_{PY}^{(0)}$ with $d_{PY}^{(0)}$.
- Aggregation of the discounted CY claim with the settlement result.
- Shifting of the resulting distribution by the deterministic value $\left(P - K - d_{CY}^{(0)} E[S_{CY}]\right)$

In the following, we will discuss the second-to-last point (aggregation of discounted CY claim with the settlement result) in more detail:

As shown above, the term $d_{CY}^{(0)}(S_{CY} - E[S_{CY}])$ is composed of normal and major claims. Essentially, there are two options for the aggregation of the normal and major claims: It is possible to model the normal claims expense S_{CY}^{KS} S^{NC}_{CY} with a lognormal distribution (expected value and variance as under 4.4.7) and to aggregate this distribution by folding it with the distribution of the major claims expense. This would result in the distribution of the CY claims. This approach is displayed in the following figure:

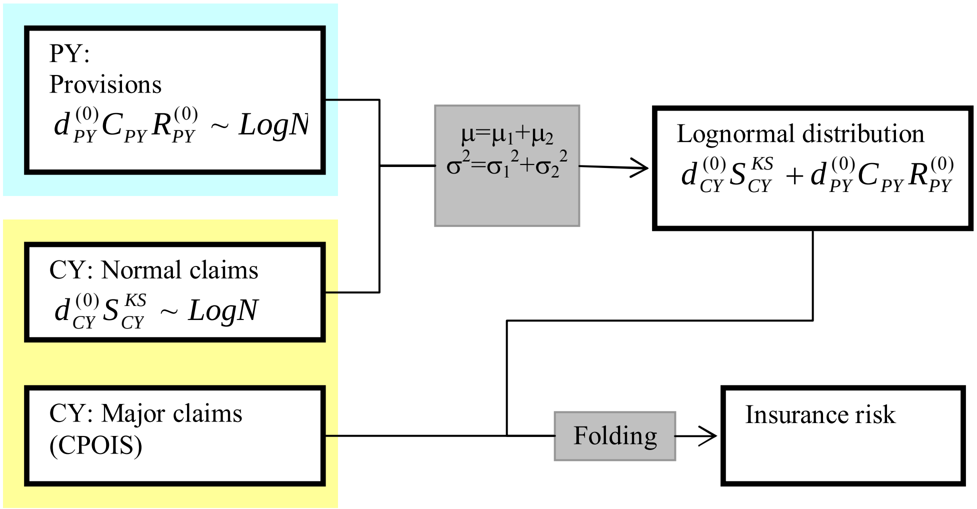


KS->NC
Figure 6: Aggregation of the insurance risks for formula (17b) or (19). Aggregation of the major and normal claims, then aggregation with the provision risk. Please note that the folding takes the discount factors into account.

Deviating from this, the standard model of the SST allows a folding operation to be omitted. This is accomplished by aggregating the normal claims expense with the uncertainty of the provisions

$$d_{CY}^{(0)} S_{CY}^{NS} + d_{PY}^{(0)} C_{PY} R_{PY}^{(0)}$$ NS->NC

This aggregation can be performed approximatively by adding expected values and variances. The SST assumes that the resulting variable has a lognormal distribution.



KS->NC
Figure 7: Aggregation of the insurance risks for formula (17b) or (19). Aggregation of the provision risks and the normal claims through the addition of moments, then aggregation with the major claims. Please note that the folding takes the discount factors into account.

In both cases, it suffices if the expected value and the variance of the annual claims expense are estimated for normal claims.

4.4.12. Reinsurance

Reinsurers are free to incorporate reinsurance as suitable when modelling claims. By modelling the major claims expense as a compound Poisson random variable, XL coverages, for instance, can easily be taken into account. Since the individual reinsurance programmes of the individual companies are too different, the SST does not impose any additional rules. Proportional coverages can also be represented easily.

If reinsurance is included in the claims modelling, premiums and costs for reinsurance must also be included in the payment flows, however, and the reinsurance scenario must additionally be included when aggregating the scenarios.

4.5. Standard model for health insurers

4.5.1. Introduction

Unlike for life and non-life insurers, the simplifying assumption is made for health insurers that the claims provisions of the health insurers do not span more than one year, but rather are used up within a year. This entails that the value of the claims provisions in the market-consistent balance sheet of the SST is not discounted. Accordingly, the claims provisions do not depend on the interest-rate curve and do not bear any interest rate risk (unlike the provisions of life and non-life insurers). The model for the market risks is therefore not an asset-liability model, but rather a pure asset model. Because of the one-year span of the provisions, the calculation of the market value margin is also omitted.

Since the provisions are independent of the interest rates, the market risks and the technical risks can be easily separated. Here, we will only describe the technical risk. Section 4.1 deals with the market risk.

The business subject to the SST described here is the health insurance business under the Insurance Contract Act. No conclusions are drawn about any business in compulsory health insurance.

4.5.2. Modelling

The values of the assets and the liability at times t_0 = beginning of the year and t_1 = end of the year are $A(0)$, $A(1)$, $L(0)$ and $L(1)$. $L(0)$ and $L(1)$ contain claims provisions and any retirement provisions.

Over the course of the year, the values of the assets and the liabilities change. The reasons for this are payment flows and changes in value. The most important payment flows include premium income P, operational and administrative costs K, the insurance benefits paid for claims S, and any dividends and coupon returns on the assets. Changes to the values of the items arise, for instance, from market value changes of the assets (change of the market price over the course of a year). We will now discuss the modelling of the values of the assets and liabilities in detail.

The value of the assets at the end of the year ($t = t_1$) is

$$A(1) = (1+R) \cdot A(0) + P - K - S\ ,$$

where the term $R \cdot A(0)$ represents the stochastic one-year performance of all assets. It includes both returns (e.g. coupon payments, dividends) and price fluctuations. From the perspective of time t_0, the performance is an unknown quantity, a random variable. It has an expected value and a standard deviation that results from the composition of the asset portfolio.

The value of the liabilities at the end of the year ($t = t_1$) is initially written as

$$L(1) = L(0) + \Delta L\ .$$

We have thereby only introduced the symbol ΔL. Inserting these quantities in $\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0)$ RBC(1)...RBC(0)... gives us:

$$= \frac{(1+R)\cdot A(0)+P-K-S-(L(0)+\Delta L)}{1+r_1^{(0)}} - A(0)+L(0)$$

$$= \frac{1}{1+r_1^{(0)}} \cdot \left((R-r_1^{(0)})\cdot A(0)+P-K-S-\Delta L+r_1^{(0)}L(0)\right)$$

Only the benefits S and the change in value R of the assets are considered stochastic variables. As a simplification, the other variables are considered to be deterministic.

It is instructive to divide the stochastic contributions into their expected value and the fluctuation around their expected value. This gives us:

$$\begin{aligned}\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0) = & \frac{(R-E[R])\cdot A(0)}{1+r_1^{(0)}} \\ & + \frac{E[R]\cdot A(0) - r_1^{(0)}(A(0)-L(0))+P-K-E[S]-\Delta L}{1+r_1^{(0)}} \\ & - \frac{S-E(S)}{1+r_1^{(0)}}\end{aligned}$$

RBC(1)...RBC(0)...

The right side consists of three contributions. Their interpretation is as follows:

- The first line shows the randomness of the market values of the assets around the expected value at the end of the year. This term is modelled with a normal distribution centred around 0.
- The second line consists of the expected results of the financial side and the insurance side.
- Finally, the third line is the uncertainty of the annual benefits around the expected annual benefits

For considering the risk, the quantities treated as deterministic must be estimated with the state of information at time t_0. These estimates are for

- the expected premium P,
- the expected operational and administrative costs K, and
- the expected change of the provisions ΔL.

The same applies to the estimate of the expected values of the quantities treated as random variables: expected annual benefits $E[S]$,
expected performance of the assets $E[R]\cdot A(0)$.

> These estimates for the expected values can be derived from the information available at time t_0 (1 January). This also applies if the calculations for the SST are not performed on 1 January of the year, but rather later in the year, which is generally the case in practice. The reason is that, for purposes of the SST, the one-year risk of the portfolio existing on 1 January is determined from the perspective of this day and is compared with the capital available on 1 January. Estimates for the expected value can be budget or planning figures for the year, if they are not wishful figures, but rather justified estimates.

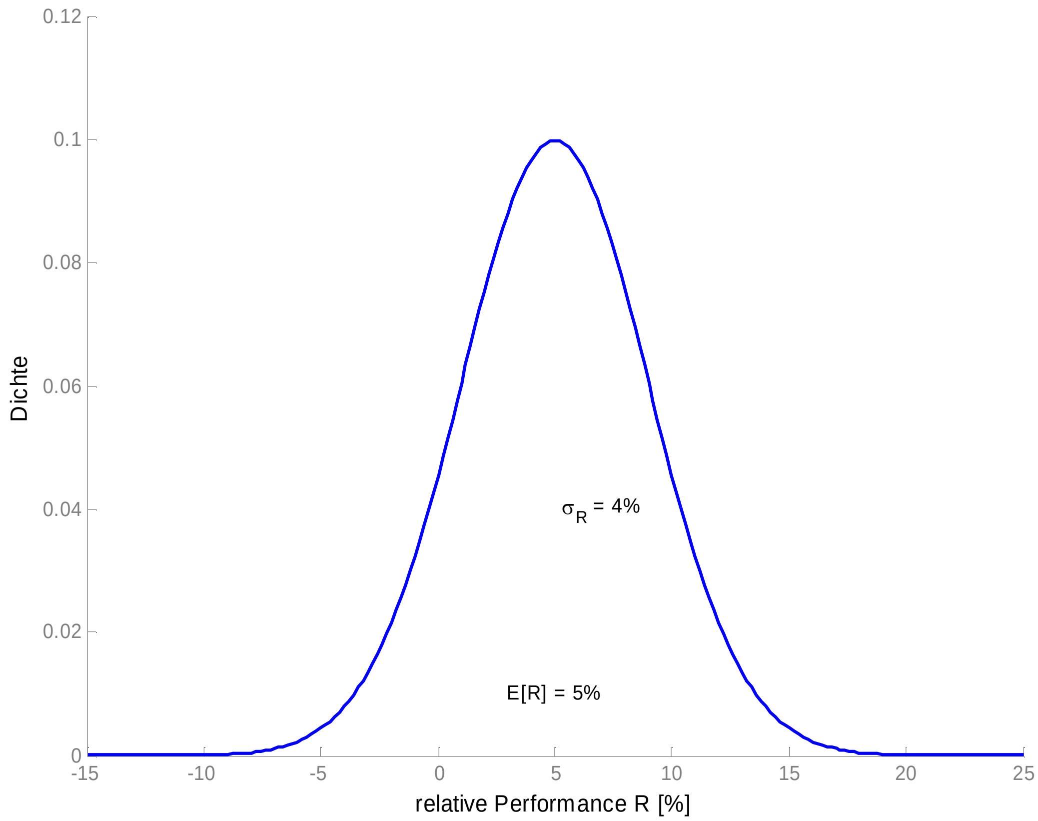
The modelling of the asset risk, i.e. the modelling of the performance of the assets $R\cdot A(0)$ contains two points. The expected performance

$$E[R]\cdot A(0),$$

which the insurer must estimate, must be distinguished from the modelling of the possible deviations of the performance from its expected value

$$R \cdot A(0) - E[R] \cdot A(0) \sim N(0, \sigma).$$

This part is a random value assumed to have a normal distribution. The calculation of its standard deviation σ is explained in section 4.1.



Density

Figure 10: Example of a distribution for performance R of the assets with an expected value of 5% and a volatility of 4%.

In the following, we will discuss the technical risk. We will describe by line of business how the quantity $S - E[S]$ (uncertainty of the annual claims around the expected annual claims) is modelled in the standard model. For purposes of simplicity, we will assume that it has a normal distribution.

4.5.3. Classification of lines of business

The standard model of the SST for health insurers distinguishes three lines of business (LoB). These are:

- E: ICA costs of care and daily allowance in individual insurance
- K: Collective daily allowance
- O: Other business operated by the health insurer

4.5.4. Insurance risk by line of business

First, we will consider the lines of business E and K. In both lines of business, the sum of the benefits S_E and S_K in one year is to be modelled. We will make the simplifying assumption that they have a normal distribution. Our goal is therefore to estimate the first two moments of the normal distribution for the annual benefits in both lines of business.

The risk in both lines of business has two causes:

- Random fluctuations of the number of cases and the variability of the amount of the individual cases. The associated risk is called random risk.
- Uncertainty in estimating the parameters, such as the expected inflation, the expected value of the number of claims, the average claims amount, etc. The associated risk is called parameter risk.

Both the random risk and the parameter risk result in a variance contribution. The total variance is the sum of these contributions.

4.5.4.1. LoB E: ICA costs of care and daily allowance in individual insurance

We use the following notation:

n_E	number of insured persons
M_E	number of claims, random variable, Poisson-distributed
$\mu_{M_E} = E[M_E]$	expected value of the number of claims
σ_{M_E}	standard deviation of the number of claims
$Y^E{}_i$	individual claims amount (i=1,..., M), random variable, i.i.d.
$\mu_{Y_E} = E[Y^E{}_i]$	expected value of the individual claims amount
σ_{Y_E}	standard deviation of the individual claims amount

Random risk

First, we will consider the random risk of the annual claims amount.
We assume that the number M_E of claims has a Poisson distribution:

$$M_E \sim Pois(\mu_{M_E})\,. \tag{48}$$

For the variance of M_E, this gives us:

$$\left(\sigma_{M_E}\right)^2 = \mu_{M_E}\,. \tag{49}$$

We do not draw any conclusions about the form of the distribution of the individual claims amount Y_i.
The annual claims amount S_E is composed of the individual claims amounts:

$$S_E = \sum_{i=1}^{M_E} Y^E{}_i\,. \tag{50}$$

For the expected value and the variance contribution of the random risk, this entails the known expressions:

$$E[S_E] = \mu_{M_E} \cdot \mu_{Y_E} \tag{51}$$

$$Var_Z(S_E) = \mu_{M_E} \cdot \sigma_{Y_E}{}^2 + \sigma_{M_E}{}^2 \cdot \mu_{Y_E}{}^2 . \tag{52}$$

Instead of the variance, we consider the variation coefficient Vko:

$$Vko_Z^2(S_E) := \frac{Var_Z(S_E)}{E^2[S_E]} = \frac{\sigma_{Y_E}{}^2}{\mu_{M_E}\mu_{Y_E}{}^2} + \frac{\sigma_{M_E}{}^2}{\mu_{M_E}{}^2} = \frac{1}{\mu_{M_E}} \cdot (Vko^2(Y_E)+1) . \tag{53}$$

$Vko(Y_E)$ is the variation coefficient of the individual claims amount, defined as $Vko(Y_E) = \frac{\sigma_{Y_E}}{\mu_{Y_E}}$.

This value is predetermined in the standard model.

Parameter risk

The contribution to the variance of S_E by the parameter risk is denoted by $Var_P(S_E)$. Instead of the variance, we can again introduce the variation coefficient $Vko_P(S_E)$:

$$Vko_P(S_E) = \frac{\sqrt{Var_P(S_E)}}{E[S_E]} . \tag{54}$$

Random risk and parameter risk combined

The addition of the parameter risk gives us

$$Vko^2(S_E) = Vko_P^2(S_E) + Vko_Z^2(S_E) = Vko_P^2(S_E) + \frac{1}{\mu_{M_E}} \cdot (Vko^2(Y_E)+1) . \tag{55}$$

Finally, the variance of the annual claims expense is

$$Var(S_E) = Vko^2(S_E) \cdot E^2[S_E] . \tag{56}$$

The statistics of the health insurers in the 2004 SST test run indicated

$$Vko(Y_E) \approx 5 .$$

The variation coefficient of the parameter risk was determined by some non-life insurers in the 2004 test run to be

$$Vko_P(S_E) = 0.0575 .$$

Given these standard values, it is possible to estimate the variability of S_E on the basis of μ_{M_E}:

$$Vko^2(S_E) = 0.0575^2 + \frac{1}{E[M_E]} \cdot \left(5^2+1\right). \tag{57}$$

This function is shown in Figure 11.

Insurers may deviate from the standard values where justified.

4.5.4.2. LoB K: Collective daily allowance

For collective daily allowance, two different models can be used to estimate the variance. These are considered equivalent by FOPI.

First method

The first method calculates the variance for collective daily allowance in the same way as for individual insurance (LoB E). It is therefore based on statements concerning the number of cases and the amount of the individual cases. The assumption for the number of cases is that it has a Poisson distribution. Similar to individual health insurance, this approach leads to

$$Vko^2(S_K) = Vko_P{}^2(S_K) + Vko_Z{}^2(S_K) = Vko_P{}^2(S_K) + \frac{1}{\mu_{M_K}} \cdot \left(Vko(Y_K) + 1\right). \tag{58}$$

Here, μ_{M_K} means the expected number of claims, and Vko(Y_K) is the variation coefficient for the individual claims amount. The evaluation of the data in the 2004 SST test run generated the following parameter values:

$$Vko(Y_K) = 2.5$$

and

$$Vko_P(S_K) = 0.0575.$$

With these standard parameters, it is again possible to estimate the variation coefficient on the basis of μ_{M_K}:

$$Vko^2(S_K) = 0.0575^2 + \frac{1}{\text{erwartete Schadenzahl}} \cdot \left(2.5^2 + 1\right). \tag{59}$$

This function is shown in Figure 11. It only depends on the expected number of claims, i.e. from a quantity that insurers can estimate easily.

Second method

In the area of collective daily allowance with a salary percentage premium, the data is generally scarcer than in the area of individual insurance. As a rule, for instance, no reliable information exists on the exact number of insured persons.

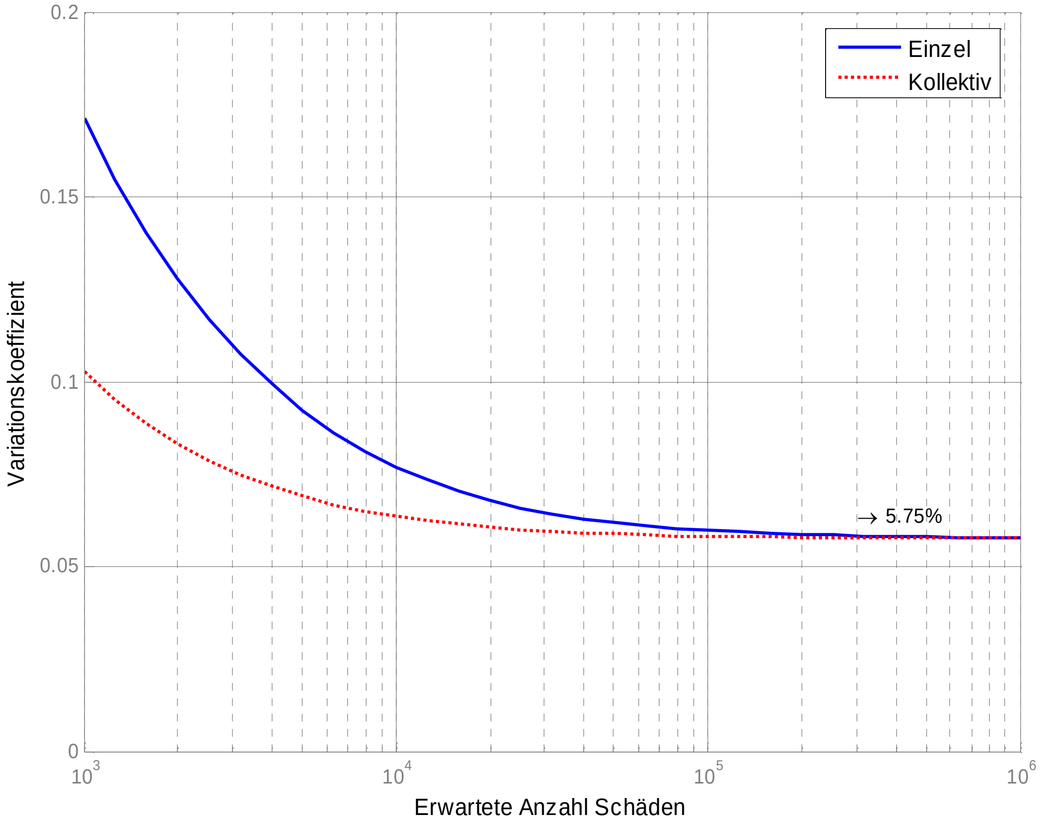
To estimate the standard deviation of the annual benefits paid as collective daily allowances, the standard deviation $\hat{\sigma}_{Hist}$ of the historically observed benefits quotas (the benefits in relation to the policy premium volume) is therefore first determined.

The estimate of the standard deviation to be used is then given by

$$\hat{\sigma} = \max(\hat{\sigma}_{Hist}, \sigma_P),$$

where $\sigma_P = 0.0575 \times E[S_K]$ is the standard deviation of the parameter risk derived by the first method. This ensures that the standard deviation used is no less than the standard deviation calculated from the parameter risk.

Against expectations, the 2004 test run showed that smaller contracts are not subject to a greater standard deviation of the benefits quota. An additional, more far-reaching data analysis did not uncover any plausible or statistically significant connection between risk and other basic parameters.



Individual
Collective
Variation coefficient
Expected number of claims

Figure 11: Variation coefficients of the annual benefits for the two lines of business "individual" and "collective" as a function of the expected number of claims in the standard model. The larger the portfolio, the more claims are to be expected, the greater the diversification, and the greater the reduction of the random risk. The parameter risk, however, cannot be diversified away, which is why both variation coefficients in large portfolios tend toward the variation coefficient (5.75%) of the parameter risk.

4.5.4.3. LoB O: Other business

The line of business "Other business" includes transactions that are not related to health insurance, but are nevertheless operated by a health insurer. These include accident insurance or household contents insurance.

By nature, these business areas harbour a technical risk. These risks are like those assumed by a typical non-life insurer, so that the treatment of the risks in the SST is analogous to non-life insurance. Accordingly, the risks of the LoB O must be quantified as in the SST for non-life insurers.

Instead of this approach, a simplified approach may be chosen, if the premium volume (after any reinsurance) of the LoB O is smaller than 10% of the total premium volume of the legal entity under consideration.
The simplified approach consists in representing the distribution of the claims expense with a normal distribution, the variance and expected value of which must be estimated by the health insurer.

5. Scenarios

5.1. Introduction

One requirement of the SST is to evaluate scenarios. These are events

- that have a very small probability of occurring, and
- that have a negative effect on the RBC.

The supervisory authority predetermines several scenarios. Insurance undertakings should supplement these with their own scenarios that reflect their own specific risk situation. If a risk described by a scenario has not already been modelled elsewhere, the evaluation of the scenarios must be incorporated in the calculation of the target capital. The SST therefore uses two types of scenarios:

- Type 1: Scenarios that must be evaluated and whose effect is aggregated with the distribution of a distribution-based model ("aggregation method"). Scenarios of this type concern risks that are not covered in the distribution-based model.

- Type 2: Scenarios that must be evaluated, but whose effect is not aggregated with the distribution-based model. Scenarios of this type concern risks that are already covered by the distribution-based model. The evaluation of the scenario can serve to support or adjust the assumptions in the distribution-based model.

For these scenarios, effects must also be taken into account that not only concern the insured claims amount. If a scenario has effects that concern the insurance business in other ways, these effects must be included in the calculation. A scenario "dirty bomb in a European city", for instance, has the consequence of direct insurance benefits, but it also has an impact upon the financial markets and the national economy. These effects must also be taken into account.

For every scenario i, the insurance undertaking must estimate the expected effect (c_i) on the risk-bearing capital. The evaluation of a scenario enables verification of whether the risk-bearing capital at the beginning of the year is sufficient under such a scenario. The scenarios of type 1, however, are not only to be used as a stress test; they also directly affect the target capital. The method to be used for the standard model is described below. To a reasonable extent, the method can be adjusted from the standard model in the direction of an internal model.

5.2. Scenarios in the standard model

The part of the standard model described so far is based on a distribution function for changes to the RBC. By including them, it is easier to capture the tail of the distribution. Attention should be paid to the fact that claims from scenarios are not already reflected by the claims in the distribution-based model.

This approach is based on the idea that the analytically modelled distribution does not adequately take into account certain extreme situations.

In the case of some of the predefined scenarios, it is possible that the effects for an insurer are positive, i.e. that they generate profit. In this event, it is permissible to include such scenarios as well. However, it would not be permissible for an insurer to formulate a scenario itself so that its evaluation would result in a profit for the insurer.

5.2.1. List of the predefined scenarios

The following table is a list of the scenarios for life, health, and non-life insurers. The formulation of the scenarios relates to the standard model. The significance of the scenarios of type 2 is explained in the preceding section 5.1.

Scenario	Probability of occurrence	Life	Non-life	Health
Industrial	0.5%		×	
Pandemic	1%	×	×	×
Accident on a works outing	0.5%		×	×
Accident: Panic in a football stadium	Type 2: not relevant for target capital.		×	×
Hail scenario	Type 2: not relevant for target capital.		×	
Disability	0.5%	×		
Daily allowance for sickness	0.5%			×
Default of the reinsurers	Depends on the reinsurance portfolio.	×	×	×
Financial distress scenario	0.5%	×	×	×
Deflation	0.1%	×	×	×
Under-provisioning	0.5%		×	×
Anti-selection for health insurers	0.5%			×
Historical market scenarios	0.1% each	×	×	×
Terrorism	0.5%	×	×	×
Longevity	0.5%	×		

5.2.1.1. Industrial scenario

The industrial scenario posits a serious accident in an industrial plant. It considers an explosion in a chemical factory. The incidents in Schweizerhalle, Seveso, and Toulouse can serve as examples.

The effects of the scenario are:

- Release of toxic gases (e.g. chlorine or dioxin). The population in the surrounding area, the residents (e.g. city of 20,000 inhabitants) are affected at z_1 = 10%, and the employees in the company (e.g. company with 500 staff members) are affected at a higher level of z_2 = 20%. Of the affected population (without company staff),
  1. y_{11} = 1% die,
  2. y_{12} = 10% become disabled, and
  3. y_{13} = 89% require hospital treatment (costs of care, e.g. smoke poisoning) but recover.

  Of the affected employees,

  4. y_{21} = 10% die,
  5. y_{22} = 30% become disabled, and
  6. y_{23} = 60% require hospital treatment (costs of care, e.g. smoke poisoning) but recover.

  Disabilities arise from possible consequences such as the escape of chlorine (damage to lungs/eyes/skin, chemical burns). Since both employees and residents are affected, the lines of business "accident insurance (UVG)" and "liability" are certainly involved.
- Deaths and injuries as a result of the explosion only affect company employees (involved lines of business: UVG, UVG-Z)
- Damage to company property (100% loss, determined by the insurance agent); involved line of business: property insurance.
- Damage to surrounding property, water pollution (long-term environmental damage), damaged vehicles and buildings (broken glass) in the surrounding area, compensation for immaterial damage (involved LoB: liability).

- Loss of income, since the factory can only partially resume manufacturing or not at all for a certain period. This results in business interruption (assumption: 4 months, 100% business interruption).
- Deaths with life insurance benefits that become payable.

5.2.1.2. Pandemic scenario

Introduction

A pandemic is an epidemic that spreads across entire countries and continents. The Greek word "pandemia" means "all people". An epidemic is a contagious, suddenly occurring and receding mass disease. The Greek word "epidemois" means "widespread among the people". In contrast, an endemic is a disease occurring continuously in certain areas (e.g. malaria, goitre). The definition of a pandemic includes that the disease is triggered by a new pathogen and that it can be transferred from human to human.

An explanation by the Federal Office of Public Health[F] is:

> An **epidemic** is the spread of a disease limited in time within a human population. Influenza is considered an epidemic in Switzerland if 100 doctor's visits indicate 1.5 suspected cases of influenza.
>
> A **pandemic** is the spread of a disease worldwide. In contrast to an epidemic, a pandemic is therefore not localized. Since the pathogen is still unknown to the human immune system, the disease spreads quickly and infects a large part of the population.

The pandemic resulting in the most deaths in the 20th century was the influenza in the years 1918/1919 ("Spanish flu"). It was caused by the H1N1 virus and killed more people than the First World War. The estimates range from 20 to 50 million deaths. Other pandemics occurred in 1957/58 ("Asian flu") and 1968/69 ("Hong Kong flu"), with about 1 million deaths each.

The pandemic scenario consists in describing the effects of a flu pandemic today. For this purpose, a study of the Federal Office of Public Health can be used. The table below describes the result of the study.

Biometric effects in Switzerland

For purposes of the scenario, the financial effect on the insurers must be determined. Potential consequential costs such as widow's and orphan's annuities must be included in the calculation. The assumptions made must be described.

The evaluation may assume that the insurer is affected according to its market share. This assumption would not be appropriate, however, if the risk exposure of special groups (e.g. employees in the health services, high-risk persons, etc.) is above or below average.

[F] "What is a flu pandemic?", available at http://www.bag.admin.ch/influenza/01120/index.html (in German)

	Kinder	Erwachsene mit normaler Gesundheit 15-49	Erwachsene mit normaler Gesundheit 50-65	Ältere	Erwachsene mit höherem Risiko 15-65	Ältere mit höherem Risiko >65	im Gesundheits-wesen tätige Personen	Total
Betroffene Bevölkerung	**1'249'000**	**3'155'000**	**1'080'000**	**700'000**	**383'000**	**328'000**	**269'000**	**7'164'000**
Anzahl Krankheitsfälle	1'001'136	2'242'890	485'603	228'701	226'314	107'163	173'252	4'465'059
	80%	71%	45%	33%	59%	33%	64%	62%
Anzahl Arztbesuche	508'549	966'972	210'059	123'902	128'886	66'497	78'093	2'082'958
	41%	31%	19%	18%	34%	20%	29%	29%
Anzahl Hospitalisierungen	2'928	13'287	1'884	2'824	8'317	2'570	1'411	33'221
	0%	0%	0%	0%	2%	1%	1%	0%
Anzahl Betttage	20'555	25'592	6'404	25'641	76'694	58'961	8'857	222'704
	2%	1%	1%	4%	20%	18%	3%	3%
Anzahl Todesfälle	4'831	10'295	3'521	3'072	4'995	14'190	1'096	42'000
	0%	0%	0%	0%	1%	4%	0%	1%
Anzahl ausfallender Arbeitstage	0	8'519'486	1'836'142	0	921'977	0	849'512	12'127'117

Children
Healthy adults 15-49
Healthy adults 50-65
Older persons
Adults at higher risk 15-65
Older persons at higher risk >65
Persons working in healthcare
Total

Affected population
Number of cases of sickness
Number of doctor's visits
Number of hospitalizations
Number of days in bed
Number of deaths
Number of lost days of work

Persons at high risk: Patients in nursing homes, persons with chronic respiratory problems, persons with immune system deficiencies, pregnant women,…

Effects on the financial markets

It is estimated that a serious pandemic would have strong effects on the global financial markets. The interest rates would drop, spreads would increase, and most currencies would be devalued against the Swiss franc. Share prices would collapse depending on the economic sector.
The following figures are drawn from [2] and [3].

Exchange rates:

USD: - 0%
EUR: - 0%
UK: - 0%
Japan: -10%
Other Asian currencies: -35%
All other emerging market currencies: -25%

Interest rates
Interest rate changes are mentioned in [3], which we use here. For Swiss and Japanese interest rates, it can be assumed that they do not become negative and that the interest rates curves for maturities

exceeding 10 years are flat. The following table is taken from tables 9 and 10 in [3]. The figures are changes in base points (bp).

Years	CHF	EUR	UK	USA	Japan
short	-37.0	-37.0	-83.0	-50.0	-38.0
1	-34.0	-34.0	-76.1	-45.8	-35.2
2	-31.0	-31.0	-69.2	-41.6	-32.4
3	-28.0	-28.0	-62.3	-37.4	-29.6
4	-25.0	-25.0	-55.4	-33.2	-26.8
5	-22.0	-22.0	-48.5	-29.0	-24.0
6	-19.0	-19.0	-41.6	-24.8	-21.2
7	-16.0	-16.0	-34.7	-20.6	-18.4
8	-13.0	-13.0	-27.8	-16.4	-15.6
9	-10.0	-10.0	-20.9	-12.2	-12.8
10	-7.0	-7.0	-14.0	-8.0	-10.0
>10	-7.0	-7.0	-14.0	-8.0	-10.0

Spread changes
For all rating classes, we assume a general increase of the interest rate spread.

AAA +75 bp
AA +100 bp
A +150 bp
BBB +200 bp
Junk +400 bp

Share prices
We assume that the prices of shares will react very differently depending on the sector. The reasoning is taken from [2].

Losers:
Transport: -50%
Tourism: -50%
Luxury Goods: -25%
Construction: -25%
Resources/Materials: -25%
Oil and Gas: -25%
Banks: -25%
Insurance and Reinsurance: -25%
Food: -25%

Winners:
Pharmaceutical: +25%

Neutral:
Consumer (non discretionary) 0%
Utilities: 0%
Telecoms and Media: +0%

Sources:

[1] The Economics of Pandemic Influenza in Switzerland, prepared by MAPI VALUES for The Swiss Federal Office of Public Health, Division of Epidemiology and Infectious Diseases, Section of Viral Diseases and Sentinel Systems, James Piercy / Adrian Miles, March 2003
[2] Avian Flu, Science, Scenarios and Stock Ideas, Citigroup, Global Portfolio Strategist, 9 March 2006

[3] Global Macroeconomic Consequences of Pandemic Influenza, Warwick J. McKibbin and Alexandra A. Sidorenko, Lowy Institute for International Policy, Sydney, February 2006

5.2.1.3. Accident insurance scenarios

The accident insurance scenario consists of one part (mass panic in a football stadium) that only has to be evaluated, and one part (accident on a works outing) that affects the target capital. Both parts are cumulated events. The difference is that the first part affects a very large number of people, but the insurer is only involved in the event in accordance with its market share. This type of cumulated event is already covered by the distribution-based model.
The second part describes an accident on a works outing. This is an event that affects the insurer 100%. This type of cumulated event is not described by the distribution-based model.

Accident scenario 1: Mass panic in a football stadium

Note: This scenario must be evaluated, but the result does not affect the target capital. The reason is that cumulated events affecting the entire market are already covered by the compound Poisson distribution for accident insurance.

The collapse of part of a stadium causes mass panic.

Assumptions:
- Number of people in the stadium: n = 10,000
- Of the n persons, x=0,5% become disabled with a degree of disability of 100%.
- Of the n persons, y=0.5% die, half of whom are female and half of whom are male.
- Of the n persons, z=24% are injured.
- This means that one quarter of all persons is affected by bodily injury: x+y+z=25%.
- The share of the insurance undertaking is given by the market share in Swiss UVG insurance.

The insurance claims are:
- Medical treatment, aids, damage to property: average expenses (not counting minor claims) CHF 20,000
- Annuities for life:
  7. Disability annuity per disabled person, with cost-of-living adjustment, annual annuity: CHF 64,000, average age: 40
  8. Widow's annuity per widow/widower with cost-of-living adjustment: annual annuity CHF 32,000, average age 38 for women, 42 for men

Additional parameters to be taken into account:
- Probability of being married at death → Collective life statistics
- Number of children entitled to annuities at death, age of these children → Collective life statistics
- Parameters and similar values for calculating the cash flows of the current annuities
- Cost-of-living adjustments: the annuity rises by a nominal 1% per year
- Interest surpluses
- Contribution premiums
- Treatment of the Cost-of-Living Fund
- Coordination with AHV (State Old Age and Survivors' Insurance)

Each insurance undertaking is affected according to its market share. Disability cases constitute the most expensive claims. Special attention is therefore paid to them in the scenario. In these cases, an annuity must generally be paid; lump-sum settlements can be ignored.

Accident scenario 2: Accident on a works outing

A bus accident, for which all passengers are insured with the insurance undertaking. For instance, this can be the works outing of a company whose employees have UVG insurance. The cause of the accident (e.g. a natural hazard) is assumed to be such that no recourse to the liability insurance of the bus company is possible.

The following assumptions are made for the scenario:

- 50 persons are on the bus.
- 25 of these persons become disabled with a degree of disability of 100%.
- The number of deaths is 15.
- 10 persons are injured.
- The average insured UVG salary is CHF 80,000 (max. CHF 106,000).
- 2 of the 50 persons have supplemental insurance with an insurance sum of CHF 5 million.

Claims:

- Medical treatment, aids, damage to property: average expenses (not counting minor claims) CHF 20,000 per person
- Annuities for life:
    1. Disability annuity per disabled person, with cost-of-living adjustment, annual annuity: CHF 64,000, average age: 40
    2. Widow's annuity per widow/widower with cost-of-living adjustment: annual annuity CHF 32,000, average age 38 for women, 42 for men

5.2.1.4. Hail scenario

Note: This scenario must be evaluated, but the result does not affect the target capital. Reason: Hail events are already covered by a compound Poisson distribution.

Four geographic footprints of hailstorms in the following four regions are given:

- Geneva
- Berne
- Neuchâtel – Aarau
- Zurich

The footprints are provided in a separate file and each consist of a list of postal codes and associated degrees of damage to motor vehicles, buildings, and contents.

5.2.1.5. Disability scenario

The disability scenario in the SST is relevant for life insurers. Two variants are available, only one of which must be evaluated:

- Increase of rate of disablement by 25% in the business year and general long-term increase of disability by 10%.
- Increase of rate of disablement by 25% in the subsequent year and average lengthening of disablement by 1 year (for persons who have already been disabled for one year).

5.2.1.6. Daily allowance for sickness

- General increase of the number of recipients of daily allowance for sickness by 25%, and
- the benefit durations d are doubled. The limitation of the benefit durations (typically to 730 days) can be taken into account.

If the limitation of the benefit duration is not explicitly taken into account in the evaluation, the scenario leads to an increase of the normal annual benefits by the factor

$$1.25 \cdot 2 = 2.5$$

5.2.1.7. Default of the reinsurers

If passive reinsurance has been incorporated into the calculation of the target capital or the determination of the best estimate provisions, then the credit risk arising from the passive reinsurance must be determined with the help of the reinsurance scenario.

The scenario considers the risk of default of the reinsurers. It assumes a situation in which the insurer is confronted with a large insurance claim. Additionally, the reinsurers are experiencing a difficult economic year, resulting in a decrease in their ratings. Numerous reinsurers default, which entails that they can no longer (fully) meet their obligations.

This causes losses for the direct insurer consisting of three components:

- The reinsurers can no longer assume the reinsured part of the occurring major claim.
- Since numerous reinsurers have defaulted, the direct insurer must purchase new coverages and pay a new premium for these coverages.
- The reinsurers can only pay the outstanding receivables of the direct insurers from old claims in part (LGD).

The probability of the scenario is the product of the probability of a market-wide downgrading ($P[\text{Downgrading}] = 10\%$) and a weighted average of the probabilities of default of the reinsurers:

$$P[\text{Szenario}] = P[\text{Downgrading}] \cdot \sum_{i}^{alleRV} \tilde{p}_i \cdot \frac{\text{Ausstand}_i + \text{Praemie}_i}{\sum_j \text{Ausstand}_j + \text{Praemie}_j}$$

Szenario -> scenario, alleRV -> allRI, Ausstand -> outstanding, Praemie -> premium

where

- $\tilde{p}_i$ is the probability of default of reinsurer i after the downgrading, and
- $\text{Ausstand}_i + \text{Praemie}_i$ outstanding + premium is the sum of the balance with reinsurer i and the ceded premium.

The value of the scenario is defined as:

$$k \cdot (\text{Brutto} - \text{Netto}) + k \cdot \sum_j \text{Praemie}_j + LGD \cdot \sum_j \text{Ausstand}_j$$

Brutto -> gross, Netto -> net, Praemie -> premium, Ausstand -> outstanding

where

- $LGD = 0.5$ is the share of the receivables in default. LDG < 1 means that the default of a counterparty does not result in a complete loss.
- $k = 0.5$ is the share of the reinsured loss that can no longer be assumed by the reinsurers.
- $\text{Brutto} - \text{Netto}$ gross – net is a measure of a major loss and defined as the maximum of
    1. difference of (gross expected shortfall) – (net expected shortfall) of the major claims distribution
    2. difference (gross scenario 1) – (net scenario 1)
    3. ...
    4. difference (gross scenario n) – (net scenario n)

Idealized probabilities of default by rating class.

Moody's			**S&P**			**AM Best**	
Aaa	0.01%		AAA	0.01%		A++/A+	0.01%
Aa1	0.02%		AA+	0.02%		A/A-	0.15%
Aa2	0.03%		AA	0.03%		B++/B+	0.65%

Aa3	0.04%	AA-	0.04%	B/B-	1.39%
A1	0.05%	A+	0.05%	C++/C+	3.64%
A2	0.07%	A	0.07%	C/C-	8.27%
A3	0.09%	A-	0.09%	D	80%
Baa1	0.21%	BBB+	0.20%		
Baa2	0.34%	BBB	0.34%		
Baa3	0.50%	BBB-	0.43%		
Ba1	0.70%	BB+	0.52%		
Ba2	0.65%	BB	1.16%		
Ba3	2.38%	BB-	2.07%		
B1	3.33%	B+	3.29%		
B2	7.14%	B	9.31%		
B3	11.97%	B-	13.15%		
Caa-C	23.65%	CCC	27.87%		

5.2.1.8. Financial distress scenario

The scenario is applicable to life and non-life insurers (including health insurers) and contains a combination of several changes to the financial environment.

- Shares, real estate, hedge funds lose value (-30%),
- Interest rates increase by 300 bp (parallel shift of all risk-free interest-rate curves in all currencies).
- 25% cancellations during one year, then normal cancellation rate.
- New business reduces by 75%.
- For life-insurers: In the collective insurance business (Federal Act on Occupational Old Age Survivors' and Invalidity Pension Fund), no surrender deduction can be made for contracts older than 5 years.

If the insurance undertaking has a rating, and this rating is higher than subinvestment quality, then the consequences effective within a year must be determined that result from the downgrading to subinvestment grade.

Subinvestment grades are:
Moody's: Ba1, Ba2, Ba3,B1, B2, B3,Caa
S&P: BB+, BB, BB-B+, B, B-, CCC

Examples of possible consequences are calling in of third-party capital by third-party capital providers or demands by clients for letters of credit.

5.2.1.9. Deflation scenario

The scenario assumes that a global deflation occurs. It is assumed that the interest rates for all currencies drop to predefined low values. At the same time, the cancellation rate drops to 0 and the probability of exercising capital options equals 10%.

5.2.1.10. Under-provisioning scenario

This scenario assumes that the claims provisions must be increased. The increase of all claims provisions is 10%. This scenario affects non-life and health insurers.

5.2.1.11. Anti-selection scenario for health insurers

This scenario assumes that a wave of cancellations by policyholders occurs before the end of the current year, resulting from strong anti-selection: all policyholders younger than 45 withdraw from the client base. This has negative consequences on the premium income and the benefits in the subsequent year. These consequences heavily depend on how the premiums are structured depending on the age of policyholders and whether financing is built on a contribution procedure or aging provisions. For instance, it is possible that the scenario entails the conversion of aging provisions, which could compensate or even over-compensate the loss of the coverage contribution of the cancelling policyholders. Second, it is also possible that the rate structure is such that the loss of coverage contributions is small.

The scenario assumes that, if a loss has occurred, a provision is formed at the end of the current year for the expected loss.

In a normal year with a full client base, the result is

$$E^N = P_{<45} - L_{<45} - \Delta V_{<45} + P_{\geq 45} - L_{\geq 45} - \Delta V_{\geq 45} - K^N$$

For a year with anti-selection (since all aging provisions already accumulated become free for clients below the age of 45 $V_{<45}$), the result is

$$E^S = 0 - 0 - 0 + V_{<45} + P_{\geq 45} - L_{\geq 45} - \Delta V_{\geq 45} - K^S$$

P, L and ΔV stand for premiums, benefits, and the change to aging provisions; K^N and K^S are the operating and administrative costs in a normal year and in a year with a reduced client base, respectively. It must be assumed that K^S also contains fixed costs that do not recede proportionally with the client base. For instance, staff size and office expenses cannot be reduced instantaneously, but only after a few months, if at all. During this time, the original costs apply.

In this case, the value of the scenario is

$$E^S - E^N = V_{<45} - (P_{<45} - L_{<45} - \Delta V_{<45} - K^N + K^S).$$

Compared with a normal year, a result arises that is lower by the amount of the premiums minus benefits and the change to the aging provisions of the cancelling policyholders, but is increased by the already accumulated aging provisions of the cancelling policyholders.

5.2.1.12. Terrorism scenario

From the set of scenarios, choose the scenario that is most likely to be triggered by a terrorist act and for which coverage is granted. The extent of the terrorism scenario is equal to the extent of scenario i.

5.2.1.13. Historical financial market scenarios

The following historical financial market scenarios are considered:

- Stock Market Crash 1987
- Nikkei Crash 1989
- European Currency Crisis 1992
- US Interest Rates 1994
- Russia / LTCM 1998
- Stock Market Crash 2000

Each of these scenarios entails the consideration of several risk factors. These factors are displayed in the SST Template and are automatically calculated in the standard model; the individual effects of the risk factors may, however, be corrected manually.

5.2.1.14. Longevity scenario for life insurers

In the longevity scenario, it is assumed that mortality decreases twice as quickly as previously assumed. It is assumed that mortality behaves according to the following formula:

$$q_{x,t} := q_{x,t_0} \cdot e^{-\lambda_x (t-t_0)}$$

In the longevity scenario, mortality behaves as follows:

$$q_{x,t} := q_{x,t_0} \cdot e^{-2\lambda_x (t-t_0)}$$

If generation tables are used that employ a different extrapolation of mortality, then these tables must be adjusted accordingly.

5.3. Combination of the distribution and the scenarios

The distribution-based model and the scenarios each take one part of all risks into account. The goal is to combine these two parts and consider the risks in a total distribution. For this purpose, we will use the aggregation method described below.

5.3.1. The method

For purposes of simplicity, we will assume that at most one scenario can occur in the year 2005, and that this scenario will only occur once at most. This approximation is acceptable, since we assume that the scenarios are rare and that the number of scenarios is low.

We define the following events:

S_k	scenario no. k with $1 \le k \le m$ occurs
S_0	none of the scenarios S_1 to S_m occur.

Furthermore, we define the following probabilities:

p_0:=P(S_0) =	probability that no scenario occurs
p_k:=P(S_k) =	probability that scenario S_k occurs ($1 \le k \le m$)

These probabilities are specified in the scenario documentation in 4.5.

The approximation above states that the scenarios are mutually exclusive. This entails that

$$p_0 = 1 - (p_1 + p_2 + \ldots + p_m), \tag{60}$$

where $p_1 + p_2 + \ldots + p_m$ is the probability that any one of the m scenarios occurs.

For each scenario S_j, the evaluation of the scenarios shows how great the effect c_j on the risk-bearing capital is:

$$c_j := E[RTK_{31.12}(\text{Szenario tritt ein}) - RTK_{31.12}(\text{Szenario tritt nicht ein})], \qquad j=1,\ldots,m$$

...= E[RBC...(scenario occurs) – RBC... (scenario does not occur)]
As a rule, the scenarios reduce the risk-bearing capital, so that c_j are negative quantities.

A year in which no scenario occurs is called a "normal year" here. In a normal year, let the distribution function of the change to the risk-bearing capital be

$$F_0(x) := P\left(\frac{RTK_{31.12.}}{1+r_1^{(0)}} - RTK_{1.1.} \le x \middle| S_0\right). \quad \text{RTK -> RBC} \tag{61}$$

This function is the result from the distribution-based model.

5.3.2. Shift of the distribution

We postulate that the distribution function under scenario S_j is

$$F_j(x) := P\left(\frac{RTK_{31.12.}}{1+r_1^{(0)}} - RTK_{1.1.} \le x \middle| S_j\right) = F_0(x - c_j), \qquad j = 1,\ldots,m\,. \quad \text{RTK->RBC} \tag{62}$$

This approach is based on the assumption that, if a scenario occurs (e.g. industrial explosion with CHF 100 million claims expenses), all possible changes to the risk-bearing capital (Δ_{RTK} RTK->RBC) will be CHF 100 million smaller than the possible Δ_{RBC} in a normal year. This assumption is not always valid: If a scenario affects other risk factors, this would not only entail a shift, but also a deformation of the distribution function. For purposes of simplicity, we will ignore such effects.

If scenario S_j occurs, the distribution of Δ_{RTK} RTK->RBC is therefore given by the distribution Δ_{RTK} RTK->RBC without a scenario, but shifted by the value c_j.

5.3.3. Aggregation

The aggregation of the scenarios and the normal year is accomplished by determining the total distribution function of the ΔRBC from the distribution functions of the scenarios and the normal year. This function is

$$F(x) = \sum_{j=0}^{m} p_j \cdot F_j(x) = \sum_{j=0}^{m} p_j \cdot F_0(x - c_j) \qquad (63)$$

and can be determined for a set of bases, since the distribution function $F_0(x)$ and therefore also the distribution function $F_j(x)$ are given numerically. Subsequently, the VaR and the expected shortfall can be determined for F(x) at certainty level α .

It can be shown that this approach generates the same distribution as the distribution of the sum of

- the continuous random variables from the distribution-based model, and
- the independent discrete random variables S with $P(S=c_i)=p_i$ for i=0,...,m.

The intuition behind this approach is to imagine that the total distribution of the Δ_{RTK} RTK->RBC can also be derived using a Monte Carlo simulation. For this purpose, a sample is drawn from the basket of the ΔRBC without scenarios, and independently a second sample is drawn from the basket of the scenarios S_0 to S_m with the values $c_{0,\ldots,}$ c_m. The total change of the RBC is the sum of the two sample values. This consideration also shows that $F(x)$ can also be calculated easily by folding the two random variables Δ_{RTK} RTK->RBC and S .

5.3.4. Double counting of risks

Risks that occur both in the distribution-based model and in the scenarios are counted double with the aggregation method, which leads to a risk assessment result that is too high.

To avoid the double counting of risks, only scenarios should be considered in the aggregation whose risks are not reflected in the distribution-based model.

Nevertheless, it still makes sense on other grounds to evaluate scenarios that result in double counting:

Scenarios are very instructive. They can be used to

- show other offices or entities the dimension of a risk.
- The result of the evaluation of a scenario can be used to better justify the representation of the risk in the distribution-based model, since it provides additional information.

5.3.5. Scenario aggregation in the case of a normal distribution

If the cumulative distribution $F_0(x)$ is discretized and represented by bases, then the values of the function $F(x)$ must also be determined at the bases.

If $F_0(x)$ is a normal distribution, then the procedure can be abbreviated; this is integrated into the Excel template for health and life insurers:

First, the α quantile (or VaR) of $F(x)$ must be determined. The value is denoted here with q. For instance, this can be done using the Newton-Raphson procedure. Second, because of (63), the expected shortfall of $F(x)$

$$ES = \frac{1}{\alpha}\int_{-\infty}^{q} x \cdot f(x)dx,$$

where $f(x)$ is the derivation of $F(x)$, can be represented as the sum over the scenarios. Namely, $f(x) = \sum_{j=0} p_j \varphi_{c_j,\sigma}(x)$, i.e.

$$ES = \frac{1}{\alpha}\sum_{j=0}^{m}\left(p_j \int_{-\infty}^{q} x \cdot \varphi_{c_j,\sigma}(x)dx \right).$$

$\varphi_{c_j,\sigma}(x)$ is the density function of the normal distribution with expected value c_j and standard deviation σ. The integral values are provided in appendix 8.6.1 as

$$\int_{-\infty}^{q} x \cdot \varphi_{c_j,\sigma}(x)dx = -\sigma^2 \cdot \varphi_{c_j,\sigma}(q) + c_j \cdot \Phi_{c_j,\sigma}(q).$$

This entails an expected shortfall of

$$ES = \frac{1}{\alpha}\sum_{j=0}^{m} p_j \left(-\sigma^2 \cdot \varphi_{c_j,\sigma}(q) + c_j \cdot \Phi_{c_j,\sigma}(q) \right)$$

The sum can be calculated with any IT tool that allows the evaluation of the cumulative normal distribution $\Phi_{\mu,\sigma}(x)$. It is helpful to bear in mind that q is not the α quantile of $\Phi_{c_j,\sigma}$, but rather of $F(x)$. For this reason, $\Phi_{c_j,\sigma}(q) \neq \alpha$.

6. Market value margin

6.1. Introduction

The market value margin (MvM) of an item is defined as the difference between the market-consistent value and the expected value of the payment flow of the item. For many financial items such as shares and bonds, the market knows the market value because these items are traded. In such cases, the MvM is implicitly contained in the price and is no longer of interest for purposes of the SST.

Technical liabilities, however, have the characteristic that their market value is generally not observable and that the expected value of the payment flow can only be estimated. For this reason, a model value for the market value margin must be determined when calculating the market value of a technical position.

If a portfolio is in run-off, then the policyholder does not bear any loss if someone else assumes the run-off risk (settlement risk). First, this is the case if the insurer bears the risk with sufficient available risk-bearing capital, or second, if an external entity (another insurer, an investor, a capital provider) assumes the portfolio or, equivalently, adds more capital. In this second case, the external entity must make risk capital available for the run-off. It will be willing to do so if it receives compensation.

The price for a technical liability is therefore composed of an amount for the expected settlement and a compensation for the associated risk. According to the definition above, this is precisely the market value margin.

The model value used for the market value margin for a portfolio containing technical liabilities is based on the assumption that the MvM is composed of capital costs or dividends. Purely mathematically speaking, these contain a risk-free share $r_1^{(0)}$ and a risk-carrying share i_{spread} (spread) on top of this, the amount of which has been fixed at 6%.

The concept of the market value margin is valid at all times. In general, we are interested in the market value and therefore the MvM at the current time t_0. In the SST, however, the value at the end of the year (t_1) is of primary significance. For this reason, we will discuss the MvM here from this perspective.

According to the reasoning above, the risk-capital provider will make risk capital K_{t_1} available at time t_1 if the provider receives a dividend $\left(r_1^{(0)} + i_{spread}\right) \cdot K_{t_1}$. The risk capital can be invested risk-free for one year, i.e. it already generates the share $r_1^{(0)} \cdot K_{t_1}$. Accordingly, it is sufficient if an additional amount $i_{spread} \cdot K_{t_1}$ is made available. This amount is taken from the basket of the market value margin. The same applies for the additional subsequent years.

It is important for the MvM to compensate the risk-assuming party for technical risks, but not for all risks assumed. For this purpose, imagine a portfolio consisting, first, of the technical liabilities, and second, of the existing instruments (assets) that replicate the liabilities to the extent possible. For a non-life portfolio, these could be government bonds that produce the expected payment flow of the liabilities.

The MvM need only compensate the risks of this portfolio. The market risks in the currently existing portfolio, in which the assets are in general composed differently than in an optimally replicating portfolio, the MvM does not need to cover, however.

Three simple examples will demonstrate this.

6.1.1. Example A

The first example consists of a share as an asset and the technical liability to pay CHF 100 in 10 years. To simplify the example, the liability has the characteristic that it does not contain any technical risk. First of all, it is clear that the discounted expected value of the liability is $100/(1+r_{10})^{10}$, i.e. the cash value of a certain payment in 10 years. Second, the same value is also equal to the market value, since the liability can be considered as a negative zero coupon bond (in other words, as a short position in a zero coupon bond). Since the discounted expected value is obviously equal to the market value, the market value margin in this case is zero.

The party buying the package does assume a market risk, namely that the share price and the interest rates change. Does the party have to be compensated for this risk with an MvM, however? The answer is no, for two reasons.

- The share can be sold, and a zero coupon bond bought in return. The resulting portfolio then consists of the zero coupon bond and the liability, a negative zero coupon bond. These cancel each other out, so no risk exists any more. There is therefore no reason to compensate the risk with an MvM.
- If the purchaser of the portfolio should decide to retain the share, then the purchaser assumes the aforementioned market risks. These are already contained in the share price and the discounted liability. First, the market value of the share already takes the risk into account that the value of the share can change. Second, the payment-in-lieu for the interest rate risk is also already contained in the discounted value of the liability.

6.1.2. Example B

Like the first example, the second example also considers a risk-free technical liability and a risk-carrying investment. In contrast to the first example, we will assume that this investment is illiquid. This means that the asset cannot be sold immediately and that the risk-assuming party is therefore forced to bear the risk for a certain time period. However, the party need not be compensated with a market value margin on the liabilities. As in the previous example, the market value of the investment already contains the compensation for the market risk.
Moreover, the market value contains a discount for the illiquidity: Two bonds with identical characteristics except for liquidity differ by market value. The value of the illiquid instrument is lower on the market than the value of the liquid instrument.

6.1.3. Example C

The third example consists of a liability, the interest rate risks of which cannot be eliminated through matching with existing financial instruments (bonds, derivatives). The cause may be that no bond exists for a payment in the far future, or that an option imbedded in the liabilities depends on the interest-rate curve without being replicable. In both cases, the liability contains an interest rate risk that cannot be avoided.
For this interest rate risk, the risk-assuming party must be compensated, since the risk is unavoidable. Since no other value contains a compensation, the compensation must be part of the MvM.

These three examples show that the MvM is a compensation for risks that are of technical origin or that are contained as market risks in the liabilities and cannot be replicated with other financial instruments.

6.2. Definition of market value margin

Based on the spread interest rate and the one-year risk capital to be provided in the individual years after t_1, the definition of the market value margin is:

$$\frac{MvM}{1+r_1^{(0)}} := i_{spread} \cdot \left(\frac{K_{t_1}}{(1+r_1^{(0)})} + \frac{K_{t_1+1yr}}{(1+r_2^{(0)})^2} + \frac{K_{t_1+2yr}}{(1+r_3^{(0)})^3} + \ldots \right).$$

Please note that the market value margin does not belong to the risk-capital provider, but rather to the policyholder. The risk-capital provider merely has the right every year to receive payment of a dividend in the amount of

$$i_{spread} \cdot K_t$$

from the basket of the market value margin.

6.3. Future one-year risks

The question arises how the individual future one-year risks K_t $(t = t_0 + 1\text{yr}, +2\text{yr}, +3\text{yr}, \ldots)$ can be calculated. Either a full risk consideration with probability distribution, i.e. practically an SST, must be performed for each of the coming years, or the K_t are approximated in an appropriate way. The simplifying assumption can be useful that the risks K_t are proportional to another quantity p_t, the progression over time of which is better known, for instance the remaining provisions. Other proportionalities are also conceivable. For life insurance, for instance, these may be the insurance sum, expected future payments upon death, expected future disability cases, etc. With such an assumption, it follows that

$$K_t = \frac{p_t \cdot K_0}{p_0}.$$

Other discussions of the market value margin are available in the two documents
"A Primer for Calculating the SST Cost of Capital Risk Margin" and
"The Swiss Experience with Market Consistent Technical Provisions - the Cost of Capital Approach".
Both can be accessed at http://www.bpv.admin.ch/themen/00506/00552/00727/.

7. References

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8. Appendix

8.1. Notations

α	Quantile level, $0 < \alpha < 1$, close to 0. Currently and most probably also in the future, $\alpha = 1\%$.
$1-\alpha$	Certainty or confidence level of the SST, close to 1.
t	Time. We denote the beginning (1 January, midnight) of the current year with t_0 and the end (31 December, just before midnight) of the same year with t_1.
RBC(t)	Risk-bearing capital (available capital) at time t.
TC	Target capital, required RBC at time t_0.
MVM	Market value margin of the liabilities, approximated by the cash value of the capital costs for future target capital of the run-off portfolio.
$(r_j^{(0)} \equiv 0, r_1^{(0)}, r_2^{(0)}, ...)$	Current curve of risk-free interest rates: risk-free returns at time t_0 for maturities of 0, 1, 2,...years.
$v_j^{(0)} = \frac{1}{(1+r_j^{(0)})^j}$	Discount factor for determining the cash value at time t_0 of a payment made at time $t_0 + j$ yr.
$(R_j^{(1)} \equiv 0, R_1^{(1)}, R_2^{(1)}, ...)$	Curve of the risk-free interest at the end of the year: risk-free returns for maturities of 0, 1, 2, ... years valid at time t_1. From the perspective of time t_0, $R_j^{(1)}$ are random variables.
$V_\mathrm{j}^{(1)} = \frac{1}{(1+R_J^{(1)})^j}$	Random variable for the discount factor for determining the cash value at time t_1 of a payment made at time $t_1 + j$ yr. From the perspective of time t_0, $\mathrm{V}_\mathrm{j}^{(1)}$ are random variables.
$A(t)$	Market value of the assets at time t, where only t_0 and t_1 occur in the SST.
$L(t)$	Discounted best estimate value of the liabilities at time t.
R_I	Random variable for the performance of the assets in the current year.
Δ_{RTK}	Difference $\frac{RTK(1)}{1+r_1^{(0)}} - RTK(0)$. Note that the difference $RTK(1) - RTK(0)$ is not used in the SST. RTK -> RBC
LoB	Line of business

8.2. Life: Target capital in a normal year

The target capital is denoted with TC and is defined as the expected shortfall (ES) of the difference of the risk-bearing capital $C(1)$ minus $C(0)$, where $C(t) = A(t) - L(t)$. Here, $A(t)$ denotes the market-consistent value of the assets at time t, and $L(t)$ denotes the best estimate of the liabilities at time t, see section 1.2. Time $t = 0$ corresponds to 31 December 2003 and $t = 1$ to 31 December 2004.

We use $Z(t) = (Z^1(t), Z^2(t), \ldots, Z^d(t))$ to denote the vector of the risk factors at time *t*. We assume that

$$C(t) = f\big(Z(t)\big)$$

with a time-invariant function f. Let $X(t) = Z(t) - Z(t-1)$. The vector $X(t)$ denotes the *change* of the risk factors between times $t-1$ and *t*.
We can therefore write

$$\begin{aligned} C(1) &= f\big(Z(1)\big) \\ C(1) - C(0) &\approx \nabla f\big(Z(0)\big) \cdot X(1) \\ &= f\big(Z(0) + Z(1) - Z(0)\big) \\ &= f\big(Z(0) + X(1)\big) \\ &\approx f\big(Z(0)\big) + \nabla f\big(Z(0)\big) \cdot X(1) \\ &= C(0) + \nabla f\big(Z(0)\big) \cdot X(1) \end{aligned}$$

Which entails:

and therefore for the target capital:

$$TC = \mathrm{ES}\big[C(1) - C(0)\big] \approx \mathrm{ES}\big[\nabla f\big(Z(0)\big) \cdot X(1)\big]$$

The smaller the volatilities $X(1)$, the better the (linear) approximation.

Let $b = \nabla f\big(Z(0)\big) = \big(\partial C(0)/\partial x^1, \ldots, \partial C(0)/\partial x^d\big)$. The quantity $\partial C(0)/\partial x^j$ denotes the relative change (sensitivity) of the risk-bearing capital $C(0)$ at time $t = 0$ per unit volatility of the risk factor x^j.

According to the SST guidelines, we assume that the vector $X(1)$ has a multivariate normal distribution with the mean $\mu = 0$ and the covariance matrix Σ. The covariance matrix Σ is given by

$$\Sigma = \Delta\ R\ \Delta$$

where $R = \left(\rho_{ij}\right)_{i,j}$ represents the correlation matrix that specifies the linear dependency structure among the risk factors. Moreover, let $\Delta = \mathrm{diag}\left(\sigma_{11}, \ldots, \sigma_{dd}\right)$ denote the diagonal matrix consisting of the standard deviations of the changes of the risk factors. The matrices *R* and Δ are predetermined by the supervisory authority.

Since the vector $X(1)$ has a multivariate normal distribution, the product $b'X(1)$ has a univariate normal distribution with mean $\mu = 0$ and variance $b'\Sigma b$. The expected shortfall and therefore the target capital can therefore be calculated explicitly as:

$$TC = \mathrm{ES}\big[\, b'X(1) \,\big] = \frac{\sqrt{b'\Sigma\, b}}{\alpha}\varphi(q_\alpha(Z))$$

where φ denotes the density function of the (univariate) standard normal distribution and $q_\alpha(Z)$ the α quantile of a random variable *Z* with a standard normal distribution. Note that for $\alpha = 0.01$, $q_\alpha(Z) = -2.3263$, $\varphi(q_\alpha(Z)) = 0.026652$ and therefore

$$\frac{\varphi(q_\alpha(Z))}{\alpha} = 2.6652$$

This approach defines the standard regime for life insurance.

8.3. Sample calculation for the market and insurance risks of a life insurer

The example below is a heavily simplified illustration of the model described above and is primarily intended to show the mechanics of the standard model. In particular, it also assumes that the translation of the statutory balance sheet into the market value balance sheet (marked-to-market) has already been performed.

8.3.1. Starting situation

The target capital is to be determined for an insurance with the following market value balance sheet. In addition to the sensitivity-driven (analytic) target capital, the scenarios "pandemic" and "disability" must also be taken into consideration. The three risk factors interest rate, shares, and cancellations are considered with the following sensitivities:

- Interest rate (+/- 1 bp)
- Shares (+/- 10%)
- Cancellations (+/- 10% of the best estimate)

8.3.2. Step 0: Determination of the market value balance sheet

	Item	Value	Duration (years)
Assets:	Shares	10	-
	Bonds	90	5
Liabilities:	Reserves	80	10
	IBNR	5	0
	Risk-bearing capital	15	

8.3.3. Step 1: Calculation of the sensitivities

Sensitivities of the risk-bearing capital

Risk factor	Value	Comments
Interest rates	+ 0.035	for +1 bp interest rate change (-4.5 loss on bonds, +8 gain on reserves, +0 for IBNR)
Shares	+ 0.1	for +1% change of the share index
Cancellations	- 0.05	for +10% change of the lapse rate, e.g. from 2% p.a. to 2.2% p.a.

"Value" is determined by the companies.

8.3.4. Step 2: Definition of the volatilities of the risk factors

This definition requires careful calibration (see appendix), since the assumption of a normal distribution is not clearly met e.g. for interest rates. For the numerical example, we assume the following liabilities, which should correspond to a 1% ES. FOPI predetermines how large these volatilities are.

Risk factor	Volatility	Volatility of the RBC in CHF
Interest rate	125 bp	4.375
Shares	25%	2.5
Cancellations	100%	-0.5

Illustration: $4.375\,\text{CHF} = 0.035\dfrac{\text{CHF}}{bp}\cdot 125bp$

8.3.5. Step 3: Determination of the variance/covariance matrix Σ:

The correlation matrix R is predefined by FOPI. In this example, it is:

	Interest rate	Shares	Cancellations
Interest rate	1	-0.25	0
Shares	-0.25	1	0
Cancellations	0	0	1

From R and the volatilities, the covariance matrix can be calculated:

$$\Sigma = \Delta\, R\, \Delta$$

where

$$\Delta = \text{diag}(125, 0.025, 1) = \begin{pmatrix} 125 & 0 & 0 \\ 0 & 0.025 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

8.3.6. Step 4: Calculation of the target capital based on the sensitivities, i.e. without scenarios

$$TC = 2.6652\cdot\sqrt{b'\,\Sigma\, b} = 4.49$$

Calculation of the (analytic) target capital with the variance-covariance approach:

where $b = (0.035, 0.1, -0.05)$

8.4. Comments on the modelling of non-life insurers

8.4.1. Classification of lines of business for non-life insurers

Number	Designation	Remarks
1	MVL	Motor vehicle liability
2	MVC	Comprehensive motor vehicle insurance, without claims arising from natural hazards
3	Property	Fire insurance Natural hazard insurance Construction insurance Business property insurance Engineering, machine insurance Theft insurance Household contents insurance Other insurance against damage to property
4	Liability	Building liability insurance Private liability insurance Business liability insurance Builder liability insurance General liability insurance
5	UVG	Compulsory occupational accident insurance Compulsory non-occupational accident insurance Voluntary UVG supplemental insurance
6	Accident w/o UVG	Individual accident insurance UVG supplemental insurance Motor vehicle passenger accident insurance Other collective accident insurance
7	Health, collective	Collective health insurance
8	Health, individual	Individual health insurance
9	Transport	Goods-in-transit insurance Comprehensive rail vehicle insurance Comprehensive water vehicle insurance Water vehicle liability insurance
10	Aviation	Comprehensive aircraft insurance Aircraft liability insurance
11	Financial and surety	Credit insurance Surety insurance Construction guarantee insurance Insurance against financial losses
12	Legal expenses	Legal expenses insurance
13	Others	Travel, tourist, traffic service insurance Epidemic insurance

Table: Definition of the modelled claims types

8.4.2. Line-of-business correlation matrix for CY normal claims

In the standard model, the following correlation matrix ($\rho_{i,j}$) applies:

	MVL	MVC	Property	Liability	UVG	Accident w/o UVG	Health, collective	Health, individual	Transport	Aviation	Finance and surety	Legal expenses	Others
MVL	1	0.5		0.25	0.25	0.25							
MVC	0.5	1	0.25										
Property		0.25	1	0.25									
Liability	0.25		0.25	1									
UVG	0.25				1	0.5	0.5						
Accident w/o UVG	0.25				0.5	1	0.5						
Health, collective					0.5	0.5	1	0.25					
Health, individual							0.25	1					
Transport									1				
Aviation										1			
Finance and surety											1		
Legal expenses												1	
Others													1

Table: Correlation matrix for normal claims

8.4.3. Variation coefficients of the parameter risk for CY normal claims

Based on an evaluation of joint statistics, the standard values for the $VK_{p,i}$ by line of business are defined as follows:

Line of business	Variation coefficient of the parameter risk
MVL	3.50%
MVC	3.50%
Property	5.00%
Liability	3.50%
UVG	3.50%
Accident w/o UVG	4.75%
Health, collective	5.75%
Health, individual	5.75%
Transport	5.00%
Aviation	5.00%
Finance and surety	5.00%
Legal expenses	5.00% (preliminary)
Others	4.50%

Table: Variation coefficients of the parameter risk

8.4.4. Variation coefficients for CY normal claims (random risk)

The table provides an overview of the standard predefined variation coefficients for calculating the random risk, depending on the major claims threshold for individual companies and lines of business (CHF 1 or 5 million).

Line of business	Variation coefficient	
	Major claims threshold 1 million	Major claims threshold 5 million
MVL	7	10
MVC	2.5	2.5
Property	5	8.
Liability	8	11
UVG	7.5	9.5
Accident w/o UVG	4.5	5.5
Health, collective	2.5	2.5
Health, individual	2.25	2.25
Transport	6.5	7
Aviation	2.5	3
Finance and surety	5	5
Others	5	5

Table 5. Variation coefficients for individual claims amounts

8.4.5. Derivation of the variation coefficient of the annual claims expense for normal claims

8.4.5.1. Parameter risk and random risk

With respect to normal claims, the annual claims S of an LoB is described with the expected value E[S] and the variation coefficient Vko(S). The variability of the annual claims can be written as a sum of contributions from

- the parameter risk and
- the random risk (stochastic risk).

The parameter risk of claims expense S is due to the variability or uncertainty of the distribution parameters. These parameters are uncertain, since either the estimate of the parameters is uncertain (no statistical basis), or since the parameters themselves change due to external circumstances from year to year. This happens in a way that essentially affects all insurers the same.
Example: The expected value of the number of road traffic accidents depends on the temperature in the summer. A hot summer causes more recreational traffic than in other years and thereby results in a higher expected value of the number of accidents. At the beginning of the year, when the expected value should be estimated, these external circumstances are not yet known. The estimate is therefore characterized by uncertainty. This uncertainty cannot be diversified away; it affects both large and small insurers.
The totality of these external circumstances und uncertainties is characterized here with the risk characteristic Θ (see also the following figure). Θ is a random variable.

The stochastic risk or random risk consists in the uncertainty of the annual claims amount, if the risk characteristic Θ (external circumstances, distribution parameters) is given. For instance, the question is how the number of road traffic accidents behaves as a random variable, given that the summer is hot.

To formally introduce the parameter risk and the random risk, we will first consider the situation of a general random variable S. Below, we will then identify S with the claims expense.

As a first step, we note that the variance of S is composed of two parts

$$Var(S) = Var\left(E\left[S|\Theta\right]\right) + E\left[Var\left(S|\Theta\right)\right],$$

the parameter risk (1st term) and the random risk (2nd term).
To explain this, we consider the right side and rewrite it

$$\begin{aligned} Var(E[S|\Theta]) + E[Var(S|\Theta)] &= E[E^2(S|\Theta)] - E^2[E(S|\Theta)] + E[E(S^2|\Theta) - E^2(S|\Theta)] \\ &= E[E(S^2|\Theta)] - E^2[E(S|\Theta)] \\ &= E[S^2] - E^2[S] \\ &= Var(S) \end{aligned}$$

to obtain the left side.

The interpretation of the terms on the right side using the example of the summer is as follows.

- Depending on the type of summer (hot, average, cool), the expected value of S differs. The first terms measures its variance. It is also a measure of the uncertainty in estimating the expected value and therefore for the parameter risk.
- The second term is an average of the variances that apply for the individual types of summer. Accordingly, we look at the fluctuations of S around the expected value (random risk).

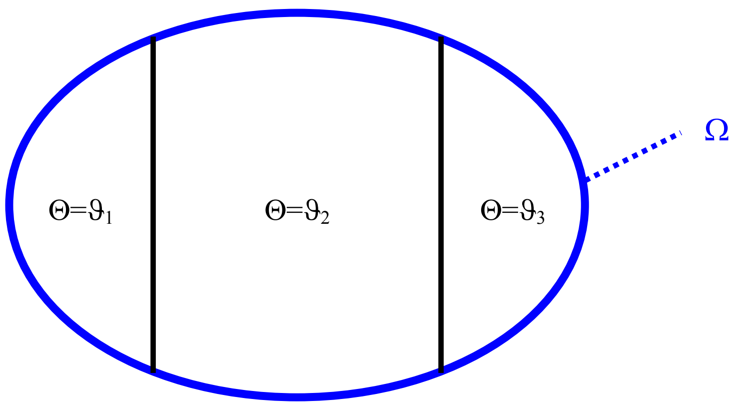
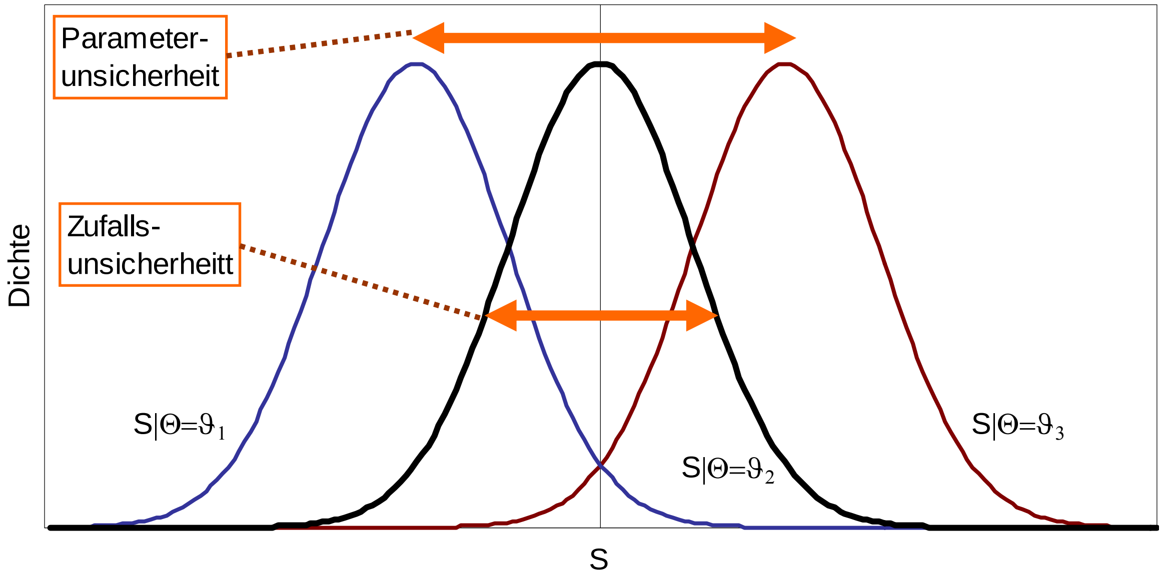


Figure: Sample illustration of probability space Ω with three possible instances of the risk characteristic Θ (e.g. Θ =ϑ_1: "hot summer", Θ =ϑ_2: "average summer", Θ =ϑ_3: "cold summer"). For each of the instances, the distribution of S is different.



Parameter uncertainty
Random uncertainty
Density

Figure: Schematic illustration of the density of S in the three states ϑ_1, ϑ_2, and ϑ_3. The random risk is the risk that arises from the random fluctuation around the expected value. The parameter risk is due to the uncertainty in estimating the expected value.

8.4.5.2. Formula (26)

We now consider S as the claims expense for normal claims in a line of business, composed of the stochastic sum

$$S = \sum_j^N Y_j$$

over the individual claims Y_j. To simplify the notation, we will omit the indices for the line of business and the designation "NC" for normal claims/minor claims.
In the preceding section, we showed that the variance of S consists of two contributions (parameter risk and random risk). For the variation coefficient, this immediately entails:

$$Vko^2(S) = \frac{Var(S)}{E^2[S]} = \frac{Var\left(E[S|\Theta]\right)}{E^2[S]} + \frac{E[Var\left(S|\Theta\right)]}{E^2[S]}.$$

We call the first term $VK_{p,i}{}^2$, the variation coefficient of S with respect to the parameter risk.
It now remains to evaluate the second term for the random risk.
With a given risk characteristic, we assume that the number of claims has a Poisson distribution

$$N|(\Theta = \vartheta) \sim Pois(\lambda(\vartheta)),$$

and that the first two moments of the individual claims amount are given by

$$E[Y_j|\Theta = \vartheta] = \mu_Y(\vartheta)$$

$$Var[Y_j | \Theta = \vartheta] = \sigma_Y^2(\vartheta)$$

The assumption of a Poisson distribution for $N|(\Theta = \vartheta)$ implies

$$E[N|\Theta = \vartheta] = Var(N|\Theta = \vartheta) = \lambda(\vartheta).$$

Given that $\Theta = \vartheta$ and given the independence of N and Y_j, the distribution of S is therefore a compound Poisson distribution. Its variance is given by the known expression:

$$\begin{aligned} Var(S|\Theta = \vartheta) &= Var(Y_j|\Theta = \vartheta) \cdot E[N|\Theta = \vartheta] + E^2[Y_j|\Theta = \vartheta] \cdot Var(Y_j|\Theta = \vartheta) \\ &= \sigma_Y^2(\vartheta) \cdot \lambda(\vartheta) + \mu_Y^2(\vartheta) \cdot \lambda(\vartheta) \end{aligned}$$

Forming the expected value over Θ gives us

$$\begin{aligned} E[Var(S|\Theta)] &= E[\sigma_Y^2(\Theta) \cdot \lambda(\Theta) + \mu_Y^2(\Theta) \cdot \lambda(\Theta)] \\ &= \sigma_Y^2 \cdot \lambda + \mu_Y^2 \cdot \lambda, \end{aligned}$$

where λ, μ_Y^2 and σ_Y^2 are the expected values $E[\lambda(\Theta)]$, $E[\mu_Y^2(\Theta)]$ und $E[\sigma_Y^2(\Theta)]$.

Note: When forming the expected value over Θ, the expected value of a product is replaced by the product of the expected values. This is only precise if the two factors ($\sigma_Y^2(\Theta)$ and $\lambda(\Theta)$) or ($\mu_Y^2(\Theta)$ and $\lambda(\Theta)$) are independent. For such independence to hold, Θ must independently affect $\mu_Y^2(\Theta)$, $\sigma_Y^2(\Theta)$ on the one hand and $\lambda(\Theta)$ on the other hand. This is achieved if we assume that Θ splits into two independent parts Θ_Y and Θ_N, and that Θ_Y only affects $\mu_Y^2(\Theta)$, $\sigma_Y^2(\Theta)$ and that Θ_N only affects $\lambda(\Theta)$.

From this, we construct the variation coefficient (with respect to the random risk)

$$\begin{aligned} Vko_Z^2(S) &= \frac{E[Var(S|\Theta)]}{E^2[S]} = \frac{\sigma_Y^2 \cdot \lambda + \mu_Y^2 \cdot \lambda}{\mu_Y^2 \cdot \lambda^2} \\ &= \frac{1}{\lambda} \cdot \left(\frac{\sigma_Y^2}{\mu_Y^2} + 1 \right) \\ &= \frac{1}{\lambda} \cdot \left(Vko^2(Y_j) + 1 \right) \end{aligned}$$

which also gives us the second term of formula (26).

8.4.6. Variation coefficients for the PY parameter risk

Variation coefficients $Vko(C_{PY} \cdot R_{PY}^{(0)}) = Vko(C_{PY})$ of the provisions with respect to the parameter risk.

Line of business	Vko
MVL	3.5%
MVC	3.5%
Property	3.0%
Liability	4.5%
UVG	3.5%
Accident w/o UVG	3.0%
Health, collective	3.0%
Health, individual	5.0%
Transport	5.0%
Aviation	5.0%
Finance and surety	5.0%
Others	5.0%

Table. Variation coefficients for the individual claims amounts

8.5. Credit risk

8.5.1. Definition of the ratings

Agency	**Rating**						
Standard&Poor's	AAA	AA+	AA	AA-	A+	A	A-
Moody's	Aaa	Aa1	Aa2	Aa3	A1	A2	A3
Fitch IBCA	AAA	AA+	AA	AA-	A+	A	A-

Agency	**Rating**								
Standard&Poor's	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
Moody's	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3
Fitch IBCA	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-

Agency	**Rating**					
Standard&Poor's	CCC+	CCC	CCC-	CC	C	D
Moody's	Caa1	Caa2	Caa3	Ca	C	
Fitch IBCA	CCC+	CCC	CCC-	CC	C	D

Source: Basel 2: Quantitative Impact Study 3: Instructions

8.6. Comments on some of the probability distributions

8.6.1. Normal distribution

The standard normal distribution has the following density and cumulative distribution function:

$$\varphi_{0,1}(x) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{x^2}{2}\right)$$

$$\Phi_{0,1}(x) = \int_{-\infty}^{x} \varphi_{0,1}(x)dx$$

The density and the cumulative distribution function of the general, univariate normal distribution are

$$\varphi_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) = \frac{1}{\sigma}\varphi_{0,1}\left(\frac{x-\mu}{\sigma}\right)$$

$$\Phi_{\mu,\sigma}(x) = \int_{-\infty}^{x} \varphi_{\mu,\sigma}(x)dx = \Phi_{0,1}\left(\frac{x-\mu}{\sigma}\right)$$

with expected value μ and standard deviation σ.
Let the random variables $X \sim N(\mu,\sigma)$ have a normal distribution. The expected shortfall

$$ES_\alpha(X) = E[X | X \leq VaR_\alpha(X)]$$

of X, where $\alpha \in (0,1)$, but often a small number close to 0, can in general not be represented in closed form. For a variable with a normal distribution, however, it can be calculated directly. We first consider the special case of a standard normal distribution $Z \sim N(0,1)$. The evaluation of the integral

$$ES_\alpha(Z) = \frac{1}{\alpha} \cdot \int_{-\infty}^{q_\alpha^{(0,1)}} y \cdot \tfrac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

results in

$$ES_\alpha(Z) = -\frac{1}{\alpha} \cdot \varphi_{0,1}(q_\alpha^{(0,1)}),$$

where $q_\alpha^{(0,1)} = \Phi_{0,1}^{-1}(\alpha)$ is the α quantile of the standard normal distribution. The expected shortfall of a quantity $X \sim N(\mu,\sigma)$ is apparently larger by the factor σ and shifted to the right by the expected value μ. We therefore obtain

$$ES_\alpha(X) = -\frac{1}{\alpha} \cdot \sigma \cdot \varphi_{0,1}(q_\alpha^{(0,1)}) + \mu .$$

Please note that this is a linear function in μ and σ.

The expected shortfall of X can also be calculated directly from the integral

$$I_{\mu,\sigma}(y)=\int_{-\infty}^{y} x\cdot\varphi_{\mu,\sigma}(x)dx=-\sigma^2\cdot\varphi_{\mu,\sigma}(y)+\mu\cdot\Phi_{\mu,\sigma}(y)\,.$$

For this purpose, $y=q_\alpha^{(\mu,\sigma)}=\Phi_{\mu,\sigma}^{-1}(\alpha)$ is specified and $\Phi_{\mu,\sigma}\left(\Phi_{\mu,\sigma}^{-1}(\alpha)\right)=\alpha$ is used. This immediately generates the same result

$$ES_\alpha(X)=\frac{1}{\alpha}I_{\mu,\sigma}(q_\alpha^{(\mu,\sigma)})=-\frac{1}{\alpha}\cdot\sigma^2\cdot\varphi_{(\mu,\sigma)}(q_\alpha^{(\mu,\sigma)})+\mu$$

as above.
As a simplification for evaluating the right side, $\varphi_{(\mu,\sigma)}(q_\alpha^{(\mu,\sigma)})=\varphi_{(0,\sigma)}(q_\alpha^{(0,\sigma)})$ can be used.
Interestingly, σ appears squared in this representation, but only in the first power further above. This is related to the fact that

$$\varphi_{(0,1)}(q_\alpha^{(0,1)})=\sigma\cdot\varphi_{(\mu,\sigma)}(q_\alpha^{(\mu,\sigma)})$$

applies in the case of density functions.

A consideration of dimensions leads to the same result:
Y is dimensionless, or Y has the dimension of a number, i.e. 1. The same applies to its density function, expected value, and standard deviation.
X, however, is not dimensionless; its dimension d is, for instance, a length or a currency, etc. The expected value, the standard deviation, and the expected shortfall also have dimension d, while the density has dimension d^{-1}. This entails that σ **must** appear in the first power in one representation and in the second power in the second representation.

8.6.2. Lognormal distribution

A random variable Y has a lognormal distribution if

$$\ln(Y/y_0)\sim N(\mu,\sigma)\,,$$

i.e. if the logarithm of Y normalized with y_0 is a quantity with a normal distribution. Occasionally, only

$$\ln(Y)\sim N(\mu,\sigma)\,.$$

is required. This definition is permissible if Y is a dimensionless quantity, i.e. a number.

Assuming that y_0 is a positive quantity, then Y is supported by the positive half-axis.
For the density distribution and cumulative distribution function of Y,

$$f_Y(y)=\frac{1}{\sqrt{2\pi\sigma^2}}\cdot\frac{1}{y}\cdot\exp\left(-\frac{(\ln(y/y_0)-\mu)^2}{2\sigma^2}\right)$$

$$F_Y(y)=\Phi_{\mu,\sigma}(\ln(y/y_0))\,,$$

where $\Phi_{\mu,\sigma}(x)$ is the cumulative distribution function of the normal distribution.

Please note that μ and σ do not denote the expected value and the standard deviation of Y. The following relationships apply between the expected value and the variance on the one hand and the parameters μ and σ on the other hand:

$$E[Y] = y_0 \cdot \exp(\mu + \tfrac{1}{2}\sigma^2)$$
$$Var(Y) = y_0^{\ 2} \cdot \exp(2\mu + \sigma^2) \cdot (\exp(\sigma^2) - 1) = E^2[Y] \cdot (\exp(\sigma^2) - 1)$$

and

$$\mu = \ln\left(\frac{E[Y]}{y_0}\right) - \tfrac{1}{2}\ln\left(\frac{Var(Y)}{E^2[Y]} + 1\right) = \ln\left(\frac{E[Y]}{y_0}\right) - \tfrac{1}{2}\sigma^2$$
$$\sigma^2 = \ln\left(\frac{Var(Y)}{E^2[Y]} + 1\right) \geq 0$$

The quantile or the value at risk with respect to quantile level α is

$$q_\alpha = y_0 \cdot \exp(\Phi^{-1}_{\mu,\sigma}(\alpha)) = y_0 \cdot \exp(\mu + \sigma \cdot \Phi^{-1}_{0,1}(\alpha)),$$

with $\Phi^{-1}_{\mu,\sigma}(\alpha)$ as the inverse function of the cumulative distribution function of the normal distribution. The expected shortfall (in the right tail of the lognormal distribution) is:

$$ES_\alpha(Y) = \frac{1}{1-\alpha} \cdot \left(1 - \Phi_{0,1}(\Phi^{-1}_{0,1}(\alpha) - \sigma)\right) \cdot E[Y].$$

Note on the derivation: The integral

$$I = \int_{q_\alpha}^{\infty} y \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot \frac{1}{y} \cdot \exp\left(-\frac{[\ln(y/y_0) - \mu]^2}{2\sigma^2}\right) dy$$

must be evaluated. By substituting $u = \ln(y/y_0)$ and completing the square, we obtain

$$I = y_0 \exp\left(\mu + \frac{\sigma^2}{2}\right) \cdot \int_{\ln(q_\alpha/y_0)}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{[u - (\mu + \sigma^2)]^2}{2\sigma^2}\right) du.$$

This integral can be considered an integral over the density of a quantity with a normal distribution with the value

$$\int_{\ln(q_\alpha/y_0)}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{[u - (\mu + \sigma^2)]^2}{2\sigma^2}\right) du = 1 - \Phi_{0,1}\left(\frac{1}{\sigma}\left(\ln(q_\alpha/y_0) - (\mu + \sigma^2)\right)\right).$$

Using the abovementioned expression $q_\alpha = y_0 \cdot \exp(\mu + \sigma \cdot \Phi^{-1}_{0,1}(\alpha))$ for the quantile, we obtain the mentioned formula for the expected shortfall.

8.6.3. Gamma distribution

We consider the two-parameter gamma distribution $Ga(\alpha, \beta)$ with the density

$$f_{\alpha,\beta}(x) = \begin{cases} \dfrac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} \exp\left(-\dfrac{x}{\beta}\right) & ,x > 0 \\ 0 & ,x \leq 0 \end{cases}$$

where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$ is the gamma distribution.

As for the lognormal distribution, the positive half-axis supports a quantity with a gamma distribution.

The parameters α and β are called shape parameter and scale parameter, respectively. In fact, β only affects the scale unit, but not the "shape" of the distribution.

Let $X \sim Ga(\alpha, \beta)$. The following relationships hold between the first two moments and the two parameters:

$$\mu = E[X] = \alpha\beta$$
$$\sigma^2 = Var[X] = \alpha\beta^2$$

and

$$\alpha = \frac{\mu^2}{\sigma^2}$$
$$\beta = \frac{\sigma^2}{\mu}$$

This immediately entails that, as expected, the variation coefficient

$$Vko(X) = \frac{\sigma}{\mu} = \frac{1}{\sqrt{\alpha}}$$

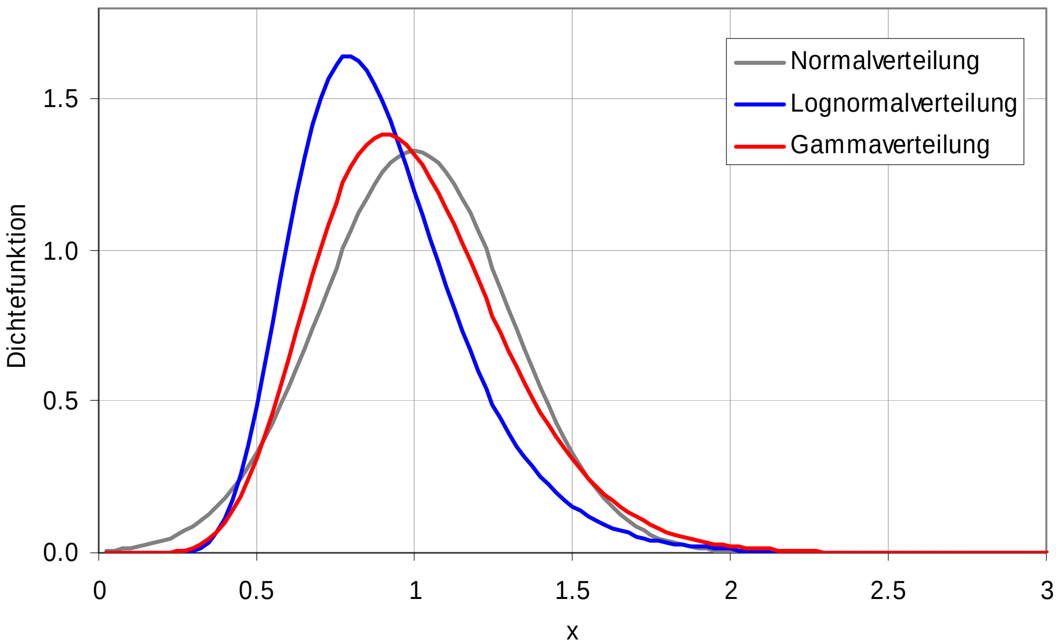
does not depend on the scale parameters.

Let $F_{\alpha,\beta}(x)$ denote the cumulative distribution function of the variable X. For the expected shortfall (in the right tail, i.e. γ close to 1) it is elementary to derive:

$$ES_{\gamma}(X) = \frac{1}{1-\gamma} \cdot \int_{q_\gamma}^{\infty} x f_{\alpha,\beta}(x) dx = \alpha\beta \cdot \frac{1 - F_{\alpha+1,\beta}(q_\gamma)}{1 - F_{\alpha,\beta}(q_\gamma)},$$

where q_γ denotes the γ quantile.

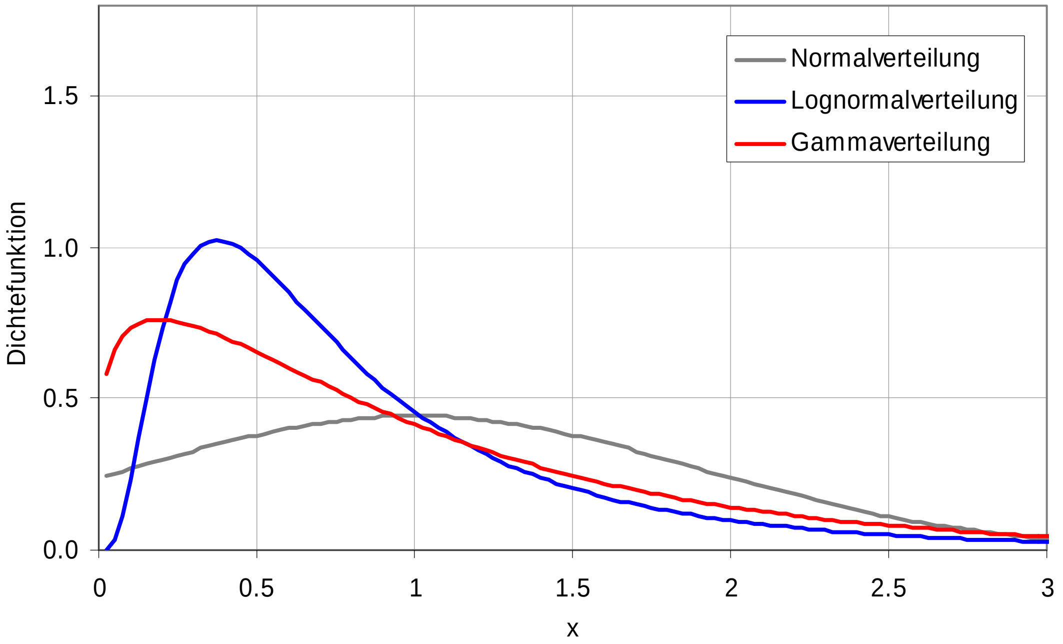
8.6.4. Comparison of normal, lognormal, and gamma distributions



Normal distribution
Lognormal distribution
Gamma distribution

Density function

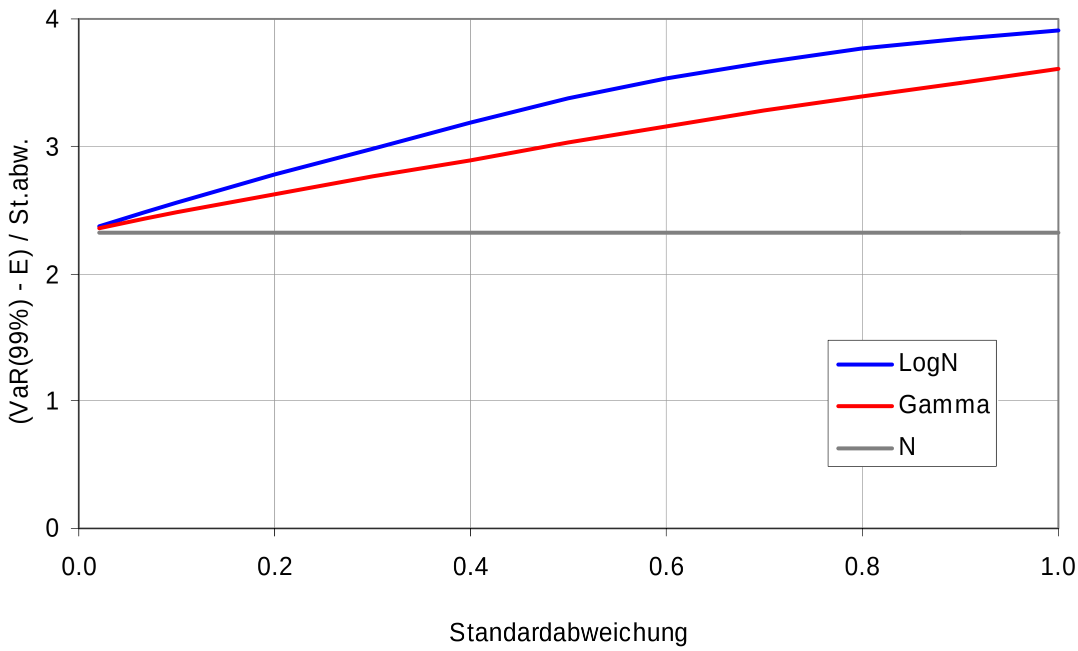
Figure 12: Density function for normal, lognormal, and gamma distributions with expected value 1 and standard deviation 0.3.



Normal distribution
Lognormal distribution
Gamma distribution

Density function

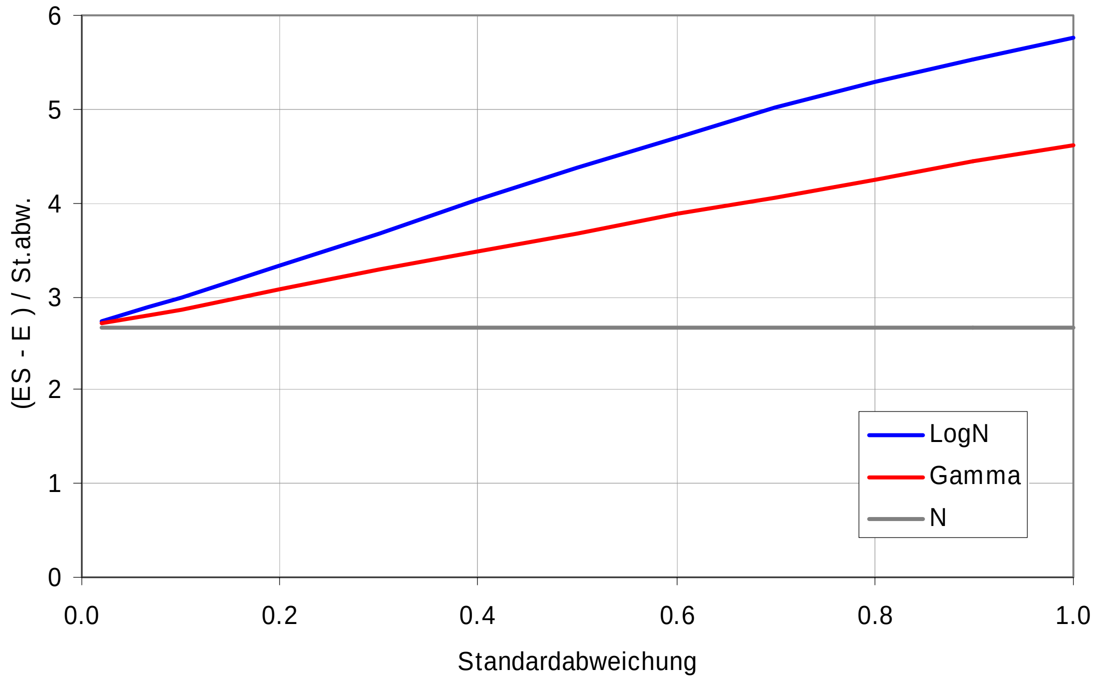
Figure 13: Density function for normal, lognormal, and gamma distributions with expected value 1 and standard deviation 0.9.



(VaR(99%) – E) / st. dev.
Standard deviation

Figure 14: Value at Risk (99%) of the normal, lognormal, and gamma distributions as a function of the standard deviation. The graph represents the difference between the VaR and

the expected value as a multiple of the standard deviation. For the normal distribution, the result is a constant value of 2.326.



(ES – E) / st. dev.
Standard deviation

Figure 15: Expected shortfall (99%) of the normal, lognormal, and gamma distributions as a function of the standard deviation. The graph represents the difference between the expected shortfall and the expected value as a multiple of the standard deviation. For the normal distribution, the result is a constant value of 2.665.

8.6.5. Pareto distribution

We consider the Pareto distribution with the cumulative distribution function

$$F(x) = \begin{cases} 0 & x < \beta \\ 1-(x/\beta)^{-\alpha} & x \geq \beta \end{cases}$$

and the distribution density function

$$f(x) = \begin{cases} 0 & x < \beta \\ \dfrac{\alpha}{\beta} \cdot (x/\beta)^{-\alpha-1} & x \geq \beta \end{cases}$$

with scale parameter β and form parameter α.
The expected value of a quantity X with a Pareto distribution exists for $\alpha > 1$ and the variance for $\alpha > 2$. These are

$$E[X] = \frac{\alpha}{\alpha - 1} \cdot \beta$$

and

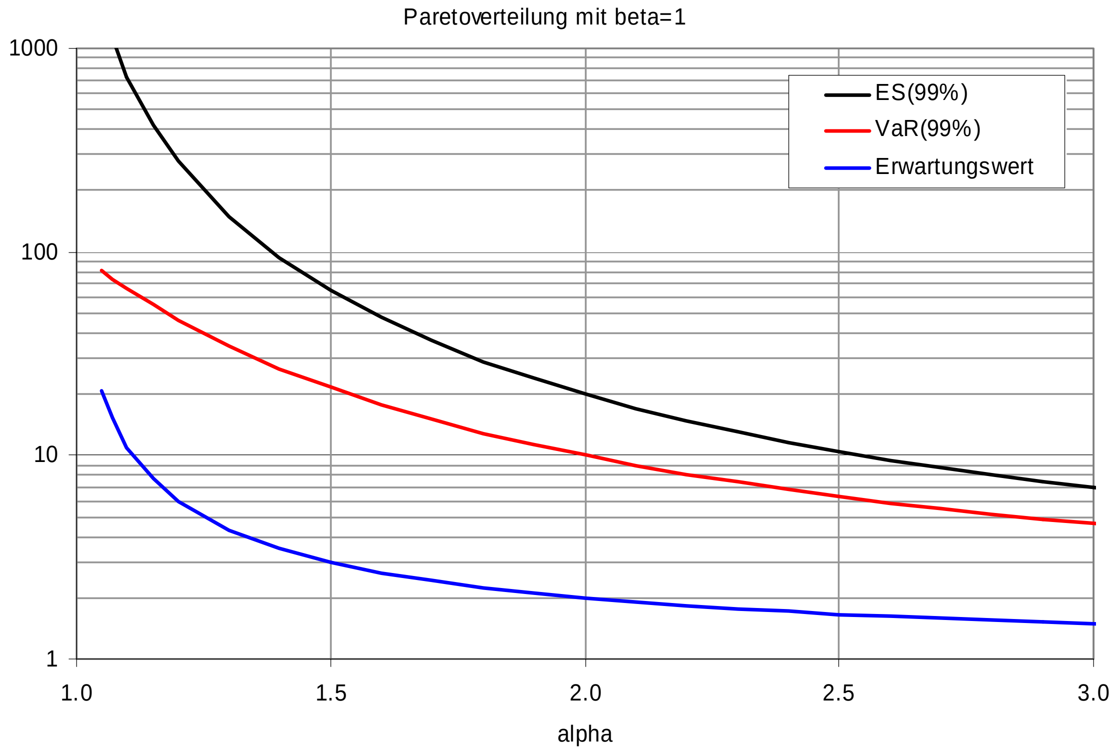
$$Var(X) = \frac{\alpha}{(\alpha-1)^2(\alpha-2)} \cdot \beta^2 .$$

The value at risk or the quantile at quantile level l is:

$$VaR_l(X) = q_l = (1-l)^{(-1/\alpha)} \cdot \beta$$

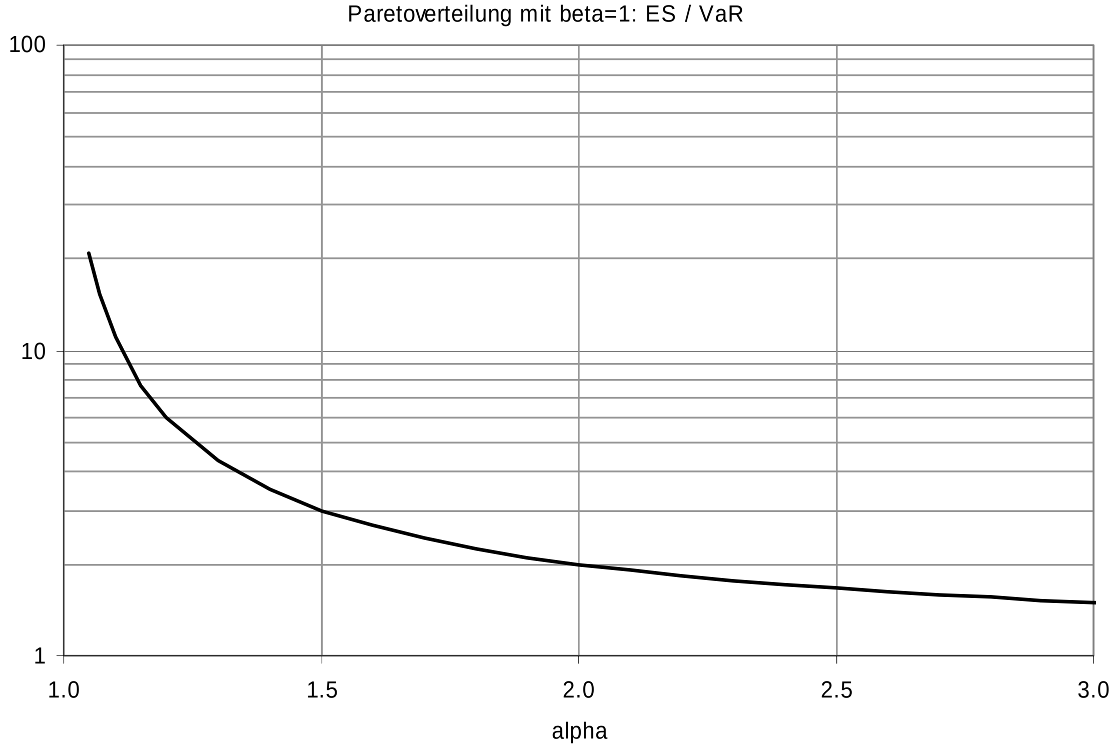
Expected shortfall at quantile level l for $\alpha > 1$:

$$\begin{aligned} ES_l(X) &= \frac{1}{1-l} \cdot \frac{\alpha}{\alpha-1} \beta^\alpha q_l^{-\alpha+1} \\ &= \frac{1}{1-l} \cdot E[X] \cdot \left(\frac{\beta}{q_l}\right)^{\alpha-1} \\ &= \frac{1}{(1-l)^{1/\alpha}} \cdot E[X] \\ &= \frac{1}{(1-l)^{1/\alpha}} \cdot \frac{\alpha}{\alpha-1} \cdot \beta \\ &= \frac{\alpha}{\alpha-1} \cdot q_l \\ &= \frac{\alpha}{\alpha-1} \cdot VaR_l(X) \end{aligned}$$



Pareto distribution with beta=1
ES(99%) VaR(99%) expected value

Figure 16: Expected value, VaR(99%), and expected shortfall (99%) for a Pareto distribution with $\beta = 1$. For small α, VaR and ES take on very high values due to the increasingly heavy tail.



Pareto distribution with beta=1: ES / VaR
Figure 17: Relationship between the expected shortfall (99%) and VaR(99%) given a Pareto distribution dependent on Pareto parameter α. For small α, the expected shortfall is significantly greater than the VaR.

8.6.6. Truncated Pareto distribution

We consider a Pareto distribution truncated at $x = \gamma$ with the cumulative distribution function

$$F(x) = \begin{cases} 0 & x < \beta \\ 1 - (x/\beta)^{-\alpha} & \beta \le x \le \gamma \\ 1 & \gamma < x \end{cases}$$

and the distribution density function

$$f(x) = \left(\frac{\gamma}{\beta}\right)^{-\alpha} \cdot \delta(x-\gamma) + \begin{cases} 0 & x < \beta \\ \dfrac{\alpha}{\beta}(x/\beta)^{-\alpha-1} & \beta \le x < \gamma, \\ 0 & \gamma \le x \end{cases}$$

where $\delta(x-\gamma)$ is the Dirac distribution of a variable with mass 1 at γ. $f(x)$ has an atom with mass $(\gamma/\beta)^{-\alpha}$ at γ. This results from the fact that the probability mass, which is above the truncation point in the normal Pareto distribution, is concentrated at γ in the truncated distribution.
For the expected value, we obtain:

$$E[X] = \begin{cases} \left(\ln(\gamma/\beta)+1\right)\cdot\beta & \alpha = 1 \\ \dfrac{\alpha}{\alpha-1}\cdot\beta\cdot\left(1-\dfrac{1}{\alpha}\left\{\dfrac{\beta}{\gamma}\right\}^{\alpha-1}\right) & \alpha \ne 1 \end{cases}$$

We shall consider the case $\alpha > 1$. The expected value of the non-truncated Pareto distribution is $\frac{\alpha}{\alpha-1}\cdot\beta$ (section 8.6.5). Accordingly, the factor $1-\frac{1}{\alpha}\left\{\frac{\beta}{\gamma}\right\}^{\alpha-1}$ represents the relationship between the expected values of the truncated and the non-truncated distribution.

8.7. Contact

Philipp Keller	philipp.keller@bpv.admin.ch	++41 31 324 93 41 ++41 79 817 07 51
Thomas Luder	thomas.luder@bpv.admin.ch	++41 31 325 01 68
Mark Stober	mark.stober@bpv.admin.ch	++41 31 323 54 19

Web http://www.bpv.admin.ch/themen/00506

Federal Office of Private Insurance FOPI

Schwanengasse 2

CH - 3001 Berne