

LZV-Tool

Technical notes

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1 Aim

Complementary health insurance business in Switzerland gives rise to dedicated long-term liabilities (*Engagements viagers EVI / Langzeitverpflichtungen LZV*) as separate position in the SST balance sheet. The LZV reflects that the insurance company cannot lapse the contract, which is a typical issue of health insurance.

FINMA provides insurance companies with an R-Tool, named LZV-Tool, to compute the LZVs.

These technical notes provide specialists with explanations on the tool and its theoretical background, for full transparency.

2 Scope

Chapter 3 provides the full theoretical background related to the functionalities in the LZV-Tool.

Chapter 4 explains the use of the tool and its underlying functions.

These technical notes are not defining the standard model health for computing the LZVs. They contain properties and functionalities broader than the standard model health, that is a particular case and is defined by the document Description technique du modèle standard SST pour l'assurance-maladie / Technische Beschreibung für das SST-Standardmodell Kranken.

3 Theoretical background

3.1 Insurance portfolio and related partition

The time t = 0 is the reference day of the SST calculation and refers usually to the balance sheet at the end of the day preceding the SST year under consideration, the current year CY. The time t = 1 denotes twelve months later. Years are indexed according to the index designing the last day of the twelve months period, so CY is numbered with index value 1. Years 0, -1,... are previous years PYs, years 2,3,... are future years FYs (sometimes called URR risk years).

The portfolio in scope of the SST is made of the business written until t = 1. As simplification the incepting date (beginning of the cover) is assumed to represent the underwriting date (beginning of the contract). The portfolio can be partitioned into the existing business, i.e. the business written until t = 0, new business, i.e. written after t = 0 until t = 1 and future business written after t = 1, the latter not being in scope of the SST. Further, it can be partitioned into earned business, i.e. the covers for treatments on PYs, and unearned business, i.e. the covers for treatments on CY and FYs. The following picture illustrates the partition.



		Treat	ment	year				
Underwriting year	t	t	t+1	t+2	t+3	t+4	t+5	
	-3							
Existing business	-2							
	-1							
	0							
New business	1							
Future business	2							
not in scope of SST								

PY= Previous treatment years; until t=0 (red line)

existing, earned

CY=Current treatment year; after t=0 until t=1 (purple line)

existing, unearned, earned at t=1

new, earned at t=1

FY=Future treatment years from t=1

existing, unearned at t=1

new, unearned at t=1

3.2 Definition of the LZV at t = 0

The long-term liabilities (*LZV*) in the SST balance sheet at time t = 0 are given by the discounted best estimate of future cash flows related to the existing unearned individual health business (*Assurance-maladie individuelle MI / Einzelkrankenversicherung EK*) at time t = 0. Only Swiss complementary health insurance in products generating long-term liabilities is in focus, meaning especially that the underlying currency is the Swiss franc (CHF). Further, no outward reinsurance is considered, which is usual for such business.

Assuming that all payments of a twelve-month time interval occur at the end of that interval, i.e. with a slight abuse of notation for avoiding integrals and measures, that for cash flows $(CF_{\tau})_{\tau>0}$

(A1)
$$CF_{s-1 < \tau \le s} = CF_s$$
 for $s = 1, 2, ...$

the **negative** of LZVs is the SST balance sheet at time t = 0 can be defined in formula as follows:

$$-\widehat{LZV_0} \coloneqq \sum_{s \ge 1} (1 + R_{0,s})^{-s} \cdot E(\widetilde{CF}_s)$$



with $\widetilde{CF}_s = \widetilde{PT}_s - LT_s - KT_s$ where:

- \widetilde{CF} denotes here $\widetilde{CF}_s^{ex,CY+FY}$, i.e. only cash flows related to the unearned existing business at time t = 0 are in focus; "ex" and "CY + FY", in superscript for indicating which sub-portfolio is considered, are omitted in the sequel unless necessary to simplify the notation.
- \widetilde{PT}_s are the total premiums (*Primes / Prämien*) paid at time s = 1, 2, ... to the company for contracts existing at time t = 0; $\widetilde{PT}_s > 0$ means an inflow to the company
- LT_s are the total benefits (*Prestations / Leistungen*) paid at time s = 1, 2, ... by the company for contracts existing at time t = 0; $LT_s > 0$ means an outflow from the company.
- KT_s are the total costs (*Frais d'administration / Verwaltungskosten*) at time s = 1, 2, ... for the company for contracts existing at time t = 0; $KT_s > 0$ means an outflow from the company.
- $R_{0,s}$ are risk free interest rates at time t = 0 for maturity times s. $R_{0,0} := 1$, for s = 1, ... 50 it is given by the is the interest risk free curve in CHF prescribed by FINMA, and for $s \ge 51$, $R_{0,s} := R_{0,50}$.
- $E(\cdot) = E(\cdot | \mathcal{F}_0)$ denotes the expectation according to the information available at time t = 0.

Note that by sign convention $\widehat{LZV_0} \ge 0$ means a real liability (expected outflow from the company), and $\widehat{LZV_0} < 0$ means an expected profit from the future cash flows, shown as a liability position in the SST balance sheet but with a negative sign. The part of LZVs generated only by benefits and costs (defined by setting $\widehat{PT}_s = 0$ above) corresponds in the vocabulary of non-life insurance to the unearned risk reserve in the balance sheet at time t=0.

It is assumed that

(A2)
$$E(\widetilde{PT}_s) = \widetilde{\Lambda}_0 \cdot E(PT_s)$$

where $\tilde{\Lambda}_0$ is a deterministic factor at time t = 0 for the expected to be unpaid but due premiums (credit default or delay) and PT_s is the premium due at time *s* according to the insurance contract conditions. The following holds:

- $(1 LGD_0) \le \tilde{\Lambda}_0 \le 1$ where LGD_0 is a value between 0 and 1 expected at time t = 0 for loss given default, typically $LGD_0 = 70\%$ in the SST.
- $\widetilde{LZV_0} \ge LZV_0$ where we define

$$-LZV_{0} = \sum_{s \ge 1} (1 + R_{0,s})^{-s} \cdot E(CF_{s})$$

with $CF_s = PT_s - LT_s - KT_s$, i.e. the LZVs including the expected unpaid premiums are higher than the LZVs ignoring them.

Comments regarding assumption (A2)

• We did not assume a relation like $E(\widetilde{CF}_s) = \widetilde{\Lambda}_0 \cdot E(CF_s)$ as alternative assumption, because they are situations where this would not lead to the economically necessary condition $\widetilde{LZV_0} \ge LZV_0$ (if cash flows are not always of the same sign) or would be undefined.



- Two main sources for not receiving due premiums are:
 - the insured does not pay to the company what is due and either an agreement is found and in the future cash flows are normal (nonetheless this step is linked with costs maybe not fully supported by the insured, reflected in $\tilde{\Lambda}_0$), or the company can lapse the contract because of non-payment of premiums but until the company has no obligations toward the insured, costs and loss payments are generated for the company.
 - the cash flows (premium income, as well as loss payments and costs) are made by another legal entity than the company itself and in an intermediary step in the book of that other legal entity. In health insurance, it allows typically for pooling the cash flows from obligatory *KVG/LAMal* and complementary *VVG/LCA* insurance, though exploited in separate entities. The rating of that other entity (if any) and the possibility of default even if the insured paid its premiums should be considered in the risk map, and as a concentration risk if applicable.
- The expected credit risk of cash inflows from benefits and costs is neglected (e.g. a health services provider who must give back to the company undue loss payments).

3.3 Valuation function at t = 0

3.3.1 Aim

The formula defining the LZVs is not directly applicable for computing them. The aim of this Section is to provide a valuation function that allows for computing the LZVs. It will be possible to give a value to the parameters of this valuation function, e.g. in the SST standard model health by prescribed values and by company estimations on own data according to prescribed estimation methods.

We begin to define a suitable partition, not depending on time of payment, of the individual health business existing at time t = 0 into contract group, age class and sex (Sections 3.3.2 and 3.3.3). In the granularity of this partition, Section 3.3.4 is devoted to the number of contracts remaining the future treatment years from those existing at time t = 0. Sections 3.3.5 to 3.3.8 derive the valuation function. Finally, adjustment factors for future years are discussed in Section 3.3.9.

3.3.2 Product groups (PG) and contract groups (CG)

The complementary health business generating long-term liabilities is partitioned into five product groups (*PG*). These PGs are further partitioned into sub-groups and tariff type, and an additional company specific split is possible if necessary, leading to contract groups (*CG*). They are shown in the following table (in the national languages French and German), up to the additional split:

Groupe de produits (PG) Produktegruppe (PG)	Sous-groupe de produits et type de tarification Produkteuntergruppe und Art der Tarifierung	Dénomination du CG <i>Bezeichnung</i> <i>der CG</i>
PG1 - Soins stationnaires semi-privés, privés et flex PG1 - Stationäre Produkte Halbprivat, Privat, Flex	.1.1 - semi-privé, âge réel .1.1 - Halbprivat, Effektivalter	CG 1.1.1
PG1 - Soins stationnaires semi-privés, privés et flex PG1 - Stationäre Produkte Halbprivat, Privat, Flex	.2.1 - privé, âge réel .2.1 - Privat, Effektivalter	CG 1.2.1
PG1 - Soins stationnaires semi-privés, privés et flex PG1 - Stationäre Produkte Halbprivat, Privat, Flex	.3.1 - flex, âge réel .3.1 - Flex, Effektivalter	CG 1.3.1



PG2 - Autres produits de soins stationnaires PG2 - Übrige stationäre Produkte	.0.1 - âge réel <i>.0.1 - Effektivalter</i>	CG 2.0.1
PG3 - Soins ambulatoires PG3 - Ambulante Produkte	.0.1 - âge réel <i>.0.1 - Effektivalter</i>	CG 3.0.1
PG4 - Soins de longue durée <i>PG4 - Langzeitpflege</i>	.0.1 - âge réel <i>.0.1 - Effektivalter</i>	CG 4.0.1
PG5 - Assurance individuelle d'indemnités journalières PG5 - Einzeltaggeld	.0.1 - âge réel <i>.0.1 - Effektivalter</i>	CG 5.0.1
PG1 - Soins stationnaires semi-privés, privés et flex PG1 - Stationäre Produkte Halbprivat, Privat, Flex	.1.2 - semi-privé, âge d'entrée .1.2 - Halbprivat, Eintrittsalter	CG 1.1.2
PG1 - Soins stationnaires semi-privés, privés et flex PG1 - Stationäre Produkte Halbprivat, Privat, Flex	.2.2 - privé, âge d'entrée .2.2 - Privat, Eintrittsalter	CG 1.2.2
PG1 - Soins stationnaires semi-privés, privés et flex PG1 - Stationäre Produkte Halbprivat, Privat, Flex	.3.2 - âge d'entrée .3.2 - Flex, Eintrittsalter	CG 1.3.2
PG2 - Autres produits de soins stationnaires PG2 - Übrige stationäre Produkte	.0.2 - âge d'entrée .0.2 - Eintrittsalter	CG 2.0.2
PG3 - Soins ambulatoires PG3 - Ambulante Produkte	.0.2 - âge d'entrée .0.2 - Eintrittsalter	CG 3.0.2
PG4 - Soins de longue durée PG4 - Langzeitpflege	.0.2 - âge d'entrée .0.2 - Eintrittsalter	CG 4.0.2
PG5 - Assurance individuelle d'indemnités journalières PG5 - Einzeltaggeld	.0.2 - âge d'entrée .0.2 - Eintrittsalter	CG 5.0.2

3.3.3 Age group and gender

The definition of an age group is based on the usage for the risk compensation in compulsory health insurance $(AOS/OKP)^{1}$.

For the treatment year YYYY, the following definitions are used:

- Age group x = YYYY year of birth = age at 31 December YYYY
 - unless YYYY year of birth > 110, then x = 110
- Index $i = i_{YYYY}$ of the age group defined by i = x + 1
- The age groups and age group indexes are hence as follows
 - x = 0, 1, ..., 109, 110
 - i = 1, 2, ..., 110, 111

For simplicity, it is assumed that an insured person can be classified in a binary manner as either male or female and this classification is referred to as gender.

3.3.4 Number of contracts at a given time

In a cell based on the above granularity *CG*, *age group* and *gender*, the number of contracts at a given time (*Effectif / Risikobestand*) is the number of covers for that time. An insured person who has two

¹ See art. 14 al. 5 LAMal as well as art. 16a al. 1, 3 et 4 LAMal



covers in this cell is counted twice. The number of contracts of several cells is the sum over these cells.

An exit for another reason than death (lapse; *résiliation / Storno*) at a given time is defined as the last day of cover at that time, i.e. cover is no longer contractually guaranteed on the following day under the current contract, for any other reason than the death of the insured.

A due emigration (*émigration due / garantierte Wechsel*) at a given time is the exercise by the insured of his right to another cover of the same type starting on the following day, without the insurer having the right to object. This is for example the case if the change from a private hospitalisation product to a flex product is to be accepted without a health check. As another example consider the switch to an equivalent product as a right from the closure of a product, including between two legal entities in the same group. A due immigration (*immigration due / garantierter Eintritt*) at a given time is the day of beginning of cover (incepting day) caused by a due emigration (the previous day).

The following notation is used. The insured's right to change cover is taken into account at the initial time (this is noted by a * below) but is no longer taken into account afterwards.

- $B_{i,0}$ = number of contracts eligible for the SST balance sheet at time t = 0 (existing contracts; *effectif existant / bestehender Bestand*) in age group x = i 1. Here and below the index *i* refers to the year CY and speaking of a number of contracts at a future time only considers contracts that existed at time t = 0.
- $B_{i,0^+}^*$ = number of contracts at time t = 0 with cover for year CY, less due emigration plus due immigration (initial number of contracts for the calculation of long-term liabilities).
- The indices j = 1, 2, ... are for the outstanding treatment years (year of cover). j = 1 denotes the current treatment year (CY) from t = 0 to t = 1.
- $B_{i,j}$ = number of contracts at 31.12 of year *j*, including due emigrations.
- B_{i,j^+} = number of contracts at 31.12 of year *j* with coverage in year *j* + 1, including due emigrations and without due immigrations (the following day).
- $\overline{B}_{i,j}$ = average number of contracts² in year *j* of policyholders counted in $B_{i,0}^*$ at time t = 0.

We have the following relations for the evolution of the number of contracts that were counted in $B_{i,0^+}^*$, due to deaths and lapses:

$$B_{i,0^+}^* \ge E(\bar{B}_{i,1}) \ge E(B_{i,1}) \ge E(B_{i,1^+}) \ge \dots \ge E(B_{i,(j-1)^+}) \ge E(\bar{B}_{i,j}) \ge E(B_{i,j}) \ge \dots$$

It will be possible to calculate the expected numbers above from the mortality and lapse probabilities as follows. We define:

² In the sense of art. 29 OAMal: "Pour le calcul de l'effectif moyen des assurés qu'il doit communiquer, l'assureur additionne les jours d'assurance de tous les assurés pour l'année considérée et divise cette somme par le nombre de jours que compte cette année."



- $q_{i,j} = E(Q_{i,j})$ with, for an insured person in CY age group x = i 1 who is counted in $B_{i,(j-1)^+}$, $Q_{i,j} = 1$ if the person dies during year *j* (the day of death is no later than 31 December of year *j*), else $Q_{i,j} = 0$. $q_{i,j}$ is the annual mortality probability.
- $s_{i,j} = E(S_{i,j})$ où, with, for an insured person in CY age group x = i 1 who is counted in $B_{i,(j-1)^+}$, $S_{i,j} = 1$ if the person lapses the contract during year *j* (the last day of cover is no later than 31 December of year *j*), due emigrations not being taken into account, else $S_{i,j} = 0$. $s_{i,j}$ is the annual lapse probability.
- $p_{i,j}$ = the survival probability in treatment year j of an insured person in CY age group x = i 1; "survival" means being still covered for treatment year j + 1:

$$p_{i,j} = E\left(\prod_{\ell=1}^{j} (1 - Q_{i,\ell}) \cdot (1 - S_{i,\ell})\right)$$

The following is assumed and allows to transform the expectation of a product into a product of expectations:

(A3) Given age index *i* in CY and treatment year *j*, the random variables $Q_{i,1}, \dots, Q_{i,\ell}, \dots, Q_{i,j}, S_{i,1}, \dots, S_{i,\ell}$ are uncorrelated

and the following is assumed for $j \ge 2$ to obtain the mortality and lapse probabilities of FYs from CY

(A4)
$$q_{i,j} = q_{i+j-1,1}$$
 and $s_{i,j} = s_{i+j-1,1}$

The relation of indexes (also used in upcoming assumptions on premium, benefits and costs) can be visualized as follows:



Reflecting that usually in complementary health insurance the covers in health insurance can be lapsed only for the 31 December, we assume

(A5)
$$B_{i,j} = q_{i,j} \cdot B_{i,(j-1)^+}$$
 for $j \ge 2$ and $B_{i,1} = q_{i,1} \cdot B_{i,0^+}^*$



By (A3) - (A5), the following recursing formulae hold for the expected number of contracts:

- $E(B_{i,0^+}^*) = B_{i,0^+}^*$, $E(B_{i,1}) = B_{i,0^+}^* \cdot (1 q_{i,1})$, $E(B_{i,1^+}) = B_{i,0^+}^* \cdot (1 q_{i,1}) \cdot (1 s_{i,1})$
- and for $j \ge 2$ with $p_{i,j} = \prod_{\ell=1}^{j} (1 q_{i+\ell-1,1}) \cdot (1 s_{i+\ell-1,1})$
 - $E(B_{i,(j-1)^+}) = B_{i,0^+}^* \cdot p_{i,j-1}$
 - $E(B_{i,j}) = E(B_{i,(j-1)^+}) \cdot (1 q_{i+j-1,1}) = B_{i,0^+}^* \cdot p_{i,j-1} \cdot (1 q_{i+j-1,1})$
 - $E(B_{i,j^+}) = E(B_{i,(j-1)^+}) \cdot (1 q_{i+j-1,1}) \cdot (1 s_{i+j-1,1}) = B_{i,0^+}^* \cdot p_{i,j^+}$

From the above we can compute the mean number of contacts, by taking the following simplifying assumption for $j \ge 1$

(A6)
$$E(\bar{B}_{i,j}) = \alpha_1 \cdot E(B_{i,(j-1)^+}) + (1 - \alpha_1) \cdot E(B_{i,j})$$
 for $j \ge 2$ and $E(\bar{B}_{i,1}) = \alpha_1 \cdot E(B^*_{i,0^+}) + (1 - \alpha_1) \cdot E(B_{i,1})$, with $\alpha_1 = 0.5$

(A6) can been rewritten as a factor from the initial number of contracts and is implemented (see R-function BesDev.f at Section 4.3.3) as follows:

$$E(\bar{B}_{i,j}) = B_{i,0^+}^* \cdot p_{i,j}^* \cdot [\alpha_1 + (1 - \alpha_1) \cdot (1 - q_{i+j-1,1})]$$

where $p_{i,1}^* = 1$ and for $j \ge 2$ $p_{i,j}^* = p_{i,j-1}$. To obtain the $p_{i,j}s$, one can rewrite their definition in a recursive algorithm, that is implemented (see R-function FactDev.f at Section 4.3.2), as follows.

- For i = 1 to 111 do: $p_{i,1} = (1 q_{i,1}) \cdot (1 s_{i,1})$
- For i = 2 to 111 ; For j = 2 to i do:
 - Nothing if $(i j + 1) \le 0$, else
 - $p_{i-j+1,j} = p_{i-j+1,j-1} \cdot (1-q_{i,1}) \cdot (1-s_{i,1})$

3.3.5 Formula for LZV, by age group and treatment year

We rewrite the formula for LZV into age group and treatment year, forgetting payment years after simplifying assumptions.

A cash flow of payments in year *s* can be decomposed according to the partition in age group and treatment year:

$$\widetilde{CF}_{s} = \sum_{i,j} \widetilde{CF}_{s}^{i,j}$$

where i = 1, ..., 111, j = 1, ..., 111 and $\widetilde{CF}_{s}^{i,j} = 0$ if $(i, j) \notin \{(i, j) | 1 \le i + j - 1 \le 111\}$

Thus, using the linearity of expectation,

$$-LZV_{0} = \sum_{s \ge 1} (1 + R_{0,s})^{-s} \cdot E(\widetilde{CF}_{s}) = \sum_{i \ge 1, j \ge 1, s \ge 1} (1 + R_{0,s})^{-s} \cdot E(\widetilde{CF}_{s}^{i,j})$$



For a treatment year *j*, we set the following simplifying assumptions:

(A7) The premiums for a treatment year *j* paid before that treatment year are accounted in $\widetilde{CF}_{j}^{i,j}$ instead of $\widetilde{CF}_{j-1}^{i,j}$

(A8)
$$\widetilde{CF}_s^{i,j} = 0 \text{ if } s \notin \{j, j+1\}$$

(A9)
$$(1 + R_{0,j+1})^{-(j+1)} \cdot E(\widetilde{CF}_{j+1}^{i,j}) \approx (1 + R_{0,j})^{-j} \cdot E(\widetilde{CF}_{j+1}^{i,j})$$

(A10) $\widetilde{CF}_{j+1}^{ex,i,j} \approx E_j(\widetilde{CF}_{j+1}^{ex,i,j})$ where $E_j(\cdot) = E(\cdot |\mathcal{F}_j)$ denotes the expectation according to the information available at time t = j.

These assumptions are justified because it is usual in complementary health insurance that almost all treatments are paid within six months from the time they are effectively done. Further, it is common that premiums are paid monthly. (A7) is expressed in word to avoid cumbersome notation. With that assumption in the LZV calculation, the premiums paid before t = 0 for current treatment year should be part of the deductions after the SST balance sheet when calculating the risk bearing capital, if this amount is not negligible for solvency.

(A10) assumes that the cash flow of treatment year *j* is deterministic at the end of treatment year *j*, i.e. at the end of that year we believe the estimate of the future cash flow for treatment year *j*. It will be useful for estimating $E(\widetilde{CF}_{s}^{i,j}) = E(\widetilde{CF}_{j}^{i,j} + \widetilde{CF}_{j+1}^{i,j})$ and it is consistent with the standard model health neglecting reserve risk. Nonetheless it could be theoretically possible to avoid this assumption because the property $E(\widetilde{CF}_{i}^{i,j} + \widetilde{CF}_{i+1}^{i,j}) = E(\widetilde{CF}_{i}^{i,j} + E_{j}(\widetilde{CF}_{i+1}^{i,j}))$ holds without.

By (A7) - (A10), we can rewrite the LZVs only with age index and treatment year:

$$-\widehat{LZV_0} = \sum_{s \ge 1} (1 + R_{0,s})^{-s} \cdot E(\widetilde{CF}_s) = \sum_{i \ge 1, j \ge 1} (1 + R_{0,j})^{-j} \cdot E(\widetilde{CF}_{i,j})$$

where $\widetilde{CF}_{i,j} = \widetilde{CF}_{j}^{i,j} + \widetilde{CF}_{j+1}^{i,j}$ denotes the cash flows of all payment years for treatment year *j* (mind the *couple* of lower indexes having replaced the single lower index for payment year).

3.3.6 Breakdown by contract group (CG) and gender

We have expressed the LZV by age group and treatment year, allowing to remove the payment year in the formula. Now we break it down by contract group (CG) and gender. This will be the required granularity for data and estimations, giving finally a formula with valuated arguments, indeed a valuation function calculating the LZV in the SST balance sheet at time t = 0. We assume:

(A11) The above assumptions (A1) - (A10) stated by age group and treatment year apply further by contract group and gender.

With (A11) and by linearity of the expectation, we can write



$$-\widetilde{LZV_0} = \sum_{CG} \sum_{i \ge 1, j \ge 1} (1 + R_{0,j})^{-j} \cdot \sum_{g \in \{\text{male}, \text{female}\}} E(\widetilde{CF}_{i,j}^{CG,g})$$

3.3.7 Formula for LZV, given CG and gender

Let be a contract group *CG* and a gender *g*, in this section. We remove the related indexes in the sequel whenever clear, e.g. we write $\widetilde{CF}_{i,j} = \widetilde{CF}_{i,j}^{CG,g}$. We aim to separate the factors related to the number of contracts (mortality, lapses) from normalised cash flow quantities by contract (premium, benefits, costs). Therefore, we write

$$\widetilde{CF}_{i,j} = \widetilde{PT}_{i,j} - LT_{i,j} - KT_{i,j} = \overline{B}_{i,j} \cdot \left(\widetilde{P}_{i,j} - L_{i,j} - K_{i,j}\right)$$

where for $1 \le i + j - 1 \le 111$

- $\tilde{P}_{i,j} = \frac{\tilde{PT}_{i,j}}{\bar{B}_{i,j}}$ is the premium by contract in treatment year *j*
- $L_{i,j} = \frac{LT_{i,j}}{\bar{B}_{i,j}}$ is the benefit by contract in treatment year *j*
- $K_{i,j} = \frac{KT_{i,j}}{\overline{B}_{i,j}}$ is the cost by contract in treatment year *j*

of an insured with CY age group x = i - 1 (if he is still insured year *j*, whence aged $x_j = x + j - 1$),

and for the remaining indexes i, j we set $\overline{B}_{i,j} = \widetilde{P}_{i,j} = L_{i,j} = K_{i,j} = 0$.

It follows that:

$$E(\widetilde{CF}_{i,j}) = E(\overline{B}_{i,j}) \cdot \{E(\widetilde{P}_{i,j}) - E(L_{i,j}) - E(K_{i,j})\} + \operatorname{Cov}(\overline{B}_{i,j}, \widetilde{P}_{i,j}) - \operatorname{Cov}(\overline{B}_{i,j}, L_{i,j}) - \operatorname{Cov}(\overline{B}_{i,j}, K_{i,j})$$

where $Cov(X, Y) = E[(X - E(X)) \cdot (Y - E(Y))] = E(X \cdot Y) - E(X) \cdot E(Y)$ defines the covariance between two random variables X and Y. All we need is to compute each of these terms to obtain a valuation of $\widehat{LZV_0}$ (the contribution related to the contact group *CG* and the gender g, of existing contracts *ex* at time t = 0), see the table below.

			CY-age index	treatment year index	Quantity to estimate						
ex	CG	g	1 <i>i</i> 111	<i>j</i> = 1,,111	$E(\bar{B}_{i,j})$	$E(\tilde{P}_{i,j})$	$E(L_{i,j})$	$E(K_{i,j})$	$\operatorname{Cov}(\bar{B}_{i,j},\tilde{P}_{i,j})$	$\operatorname{Cov}(\bar{B}_{i,j}, L_{i,j})$	$\operatorname{Cov}(\bar{B}_{i,j}, K_{i,j})$



3.3.7.1 Formula for $E(\overline{B}_{i,j})$

Section 3.3.4 applied to *CG* and *g* provides the value of $E(\overline{B}_{i,i})$ from

- CY probabilities of mortality $q_{i,1}$
- CY probabilities of lapse s_{i,1}
- the number of contracts $B_{i,0^+}^*$

3.3.7.2 Formulae for $E(\tilde{P}_{i,j})$, $E(L_{i,j})$ and $E(K_{i,j})$

We assume the relation $E(\tilde{P}_{i,j}) = \tilde{\Lambda}_0 \cdot E(P_{i,j})$ where $\tilde{\Lambda}_0$ is a value for the whole product group *PG* underlying *CG*, *g* (see assumptions (A2) and (A11)).

Following the idea of the recursion for $E(\overline{B}_{i,j})$, we set assumptions allowing to compute the above quantities for $j \ge 2$ from

- $E(P_{i,1})$
- $E(L_{i,1})$
- $E(K_{i,1})$

Thus, we assume for $j \ge 2$

- (A12) If $i + j 1 \le i_{\text{threshold}} E(P_{i,j}) = \gamma_j^P \cdot E(P_{i+j-1,1})$, else $E(P_{i,j}) = \gamma_j^P \cdot E(P_{i,[\max(i_{\text{threshold}}-i,0)+1]})$ where $i_{\text{threshold}} = x_{\text{threshold}}^P + 1$ with $x_{\text{threshold}}^P$ being the real age group from which the premium is constant, and where γ_j^P is an adjustment factor for the future year *j*.
- (A13) If $i + j 1 \le i_{\text{threshold}} E(L_{i,j}) = \gamma_j^L \cdot E(L_{i+j-1,1})$, else $E(L_{i,j}) = \gamma_j^L \cdot E(P_{i,[\max(i_{\text{threshold}}-i,0)+1]})$ where $i_{\text{threshold}} = x_{\text{threshold}}^L + 1$ with $x_{\text{threshold}}^L$ being the real age group from which the benefits are constant, and where γ_j^L is an adjustment factor for the future year *j*.
- (A14) If $i + j 1 \le i_{\text{threshold}} E(K_{i,j}) = \gamma_j^K \cdot E(K_{i+j-1,1})$, else $E(K_{i,j}) = \gamma_j^K \cdot E(K_{i,[\max(i_{\text{threshold}}-i,0)+1]})$ where $i_{\text{threshold}} = x_{\text{threshold}}^K + 1$ with $x_{\text{threshold}}^K$ being the real age group from which the costs are constant, and where γ_j^K is an adjustment factor for the future year j.

Comments regarding assumptions (A12) – (A14)

- $E(P_{i,1})$, $E(L_{i,1})$ and $E(K_{i,1})$ already contain the inflation of CY compared to PYs.
- The factors γ_j^P , γ_j^L and γ_j^K for $j \ge 2$ allows in particular for representing the premium cap and the future non-structural inflation, see section 3.3.9 later. Setting them to one for the time being, we have:



- (A12) implies that for $i + j 1 > i_{\text{threshold}}$: $E(P_{i,j}) = E(P_{i,(i_{\text{threshold}}-i)+1}) = E(P_{i_{\text{threshold}},1})$ if $i \le i_{\text{threshold}}$ else $E(P_{i,j}) = E(P_{i,1})$. If $x_{threshold}^P = 110$, this assumption is simply $E(P_{i,j}) = E(P_{i+j-1,1})$, reflecting a real age tariff. For an entry age tariff only for adults (but real age tariff for children and young people) typically $x_{threshold}^P = 26$. The assumption says implicitly that the mix of entry ages for a CY real age index *i* today will by the same at time *j* for insured with CY real age i - j + 1
- The above formulae for premiums can be easily understood by visualizing on the triangle of ages and treatment years. With *i*^{*} = *i*_{threshold}, say *i*^{*} = 27, the blue cells show the condition for a real age tariff, say premium at age 20 if age 14 in CY, the hell green cells show entry age premium after age 26 if age lower than 26 in CY, and the dark green cell show entry age premium in the future if real age greater than 26 in CY.



This allows to rewrite (A12) in form of a recursive algorithm, that is implemented (see R-function QUDev.f at Section 4.3.1), as follows. The idea is to set the first column as the curve of premium by age given in input, and then to populate the diagonals, first the second upper left diagonal from left to right, then the third diagonal from left to right, etc. The output will be a matrix QU of expected premiums by contract (QU is for *Quantity* by *Unit* of exposure).

- Initialise *QU* as a 111 x 111 matrix with entries 0
- For i = 1 to 111 do: $QU_{i,1} = E(P_{i,1})$
- For i = 2 to 111; For j = 2 to i do:
 - Nothing if $(i j + 1) \le 0$, else
 - if $(i j + 1) \le (x_{Threshold} + 1)$ do: $QU_{i-j+1,j} = QU_{i-j+2,j-1}$
 - else do: $QU_{i-j+1,j} = QU_{i-j+1,j-1}$
- (A13) with $x_{threshold}^{L} = 110$ is simply $E(L_{i,j}) = E(L_{i+j-1,1})$, reflecting the curve of benefits by real age until the last age considered. Typically, the alternative choice $x_{threshold,L} = 91$ allows for making the benefits by contract constant for old insured.
- (A14) is usually used with $x_{threshold}^{K} = 110$, reducing the assumption to $E(K_{i,j}) = E(K_{i+j-1,1})$.



3.3.7.3 Formulae for covariance terms

We set as simplifying assumptions

(A15) $\overline{B}_{i,j}$ and $P_{i,j}$ are uncorrelated

(A16) $\overline{B}_{i,j}$ and $L_{i,j}$ are uncorrelated

(A17) $\overline{B}_{i,j}$ and $K_{i,j}$ are uncorrelated

This implies

$$\operatorname{Cov}(\overline{B}_{i,j}, \widetilde{P}_{i,j}) = \operatorname{Cov}(\overline{B}_{i,j}, L_{i,j}) = \operatorname{Cov}(\overline{B}_{i,j}, K_{i,j}) = 0$$

3.3.8 Summary

Given a contract group *CG* and a gender *g* of existing contacts ex at time t = 0 generating LZV in the SST balance sheet, and assuming with the above notations

- Assumptions (A1) (A17) and
- the following values available
 - values related to CY

			CY-age index	e Estimated quantities and related parameters									
ex	CG	g	1 i 	$B_{i,0}^{*}$ +	$q_{i,1}$	<i>S</i> _{<i>i</i>,1}	$\widetilde{\Lambda}_0$	$E(P_{i,1})$	$x_{threshold}^{P}$	$E(L_{i,1})$	$x_{threshold}^{L}$	$E(K_{i,1})$	$x^{K}_{threshold}$

- factors γ_j^P , γ_j^L and γ_j^K related to the future treatment years j = 2,3,...
- risk free interest rates $R_{0,j}$ for the currency CHF and maturities j = 1, 2, ...

the negative of the best estimate value of the long-term liabilities is given by the formula

$$-\widetilde{LZV_0} = \sum_{i\geq 1,j\geq 1} (1+R_{0,j})^{-j} \cdot E(\overline{B}_{i,j}) \cdot \{\widetilde{\Lambda}_0 \cdot E(P_{i,j}) - E(L_{i,j}) - E(K_{i,j})\}$$

where

$$E(\bar{B}_{i,j}) = [\alpha_1 + (1 - \alpha_1) \cdot (1 - q_{i+j-1,1})] \cdot p_{i,j-1} \cdot B^*_{i,0^+}$$

with $\alpha_1 = 0.5$, $p_{i,0} = 1$ and $p_{i,j-1} = \prod_{\ell=1}^{j-1} (1 - q_{i+\ell-1,1}) \cdot (1 - s_{i+\ell-1,1})$ for $j \ge 2$



and for $j \ge 2$ and $1 \le i + j - 1 \le 111$

If $i + j - 1 \le i_{\text{threshold}} \quad E(U_{i,j}) = \gamma_j^U \cdot E(U_{i+j-1,1}), \qquad \text{else } E(U_{i,j}) = \gamma_j^U \cdot E(U_{i,[\max(i_{\text{threshold}}-i,0)+1]})$

with U denoting one of the symbols P, L or K and $i_{\text{threshold}} = x_{\text{threshold}}^U + 1$.

For a real age tariff with given premium cap factors $\gamma_i^{PC} \in (0, 1]$,

by setting $\tilde{\Lambda}_0 = 1$, $x_{threshold}^P = 110$, $x_{threshold}^L = 110$, $x_{threshold}^K = 110$, $\gamma_j^P = \gamma_j^{PC}$, $\gamma_j^L = 1$, $\gamma_j^K = 1$ the formula is simplified in

$$-LZV_{0} = \sum_{i \ge 1, j \ge 1} \left(1 + R_{0,j} \right)^{-j} \cdot E(\bar{B}_{i,j}) \cdot \left\{ \gamma_{j}^{PC} \cdot E(P_{i+j-1,1}) - E(L_{i+j-1,1}) - E(K_{i+j-1,1}) \right\}$$

This simplified formula is assumed in the standard model health and designed for real age tariffs.

The above formula (general case, not only real age tariffs) has been derived for a contract group *CG* and a sex *g*, removing *CG* and *g* in the notation. By linearity of the expectation the total LZV_0 is the SST balance sheet at time t = 0 is obtained by summing up the terms for each *CG* and *g*:

$$\widehat{LZV_0}^{ex} = \sum_{CG} \sum_{g \in \{male, female\}} \widehat{LZV_0}^{ex, CG, g}$$

3.3.9 Premium cap and non-structural inflation

In this section we discuss the factors γ_j^P , γ_j^L and γ_j^K for adjusting future treatment years $j \ge 2$ from CY estimations of premiums, benefits and costs, see assumptions (A12) - (A14) respectively. We aim to use these factors to represent the non-structural inflation and the premium cap.

The non-structural inflation represents the inflation not due to a change in the structure of insured persons, typically treatment costs higher because of new medicines, whereas the structural inflation would e.g. result from healthy people over-lapsing. In complementary health insurance, the premiums can be adapted to compensate the non-structural inflation and such increase in tariffs is subject to approval. The increase is usually not allowed if it aims to cover the structural inflation, that is in the risk of the insurer.

The premium cap reflects that long-term excessive return is not allowed (it could give rise to a premium reduction) in the sense of the policyholder protection. Such protection exists, because in health business non-healthy people cannot lapse to change for a competitor. We present formulae for γ_j^P , γ_j^L and γ_i^K reflecting the non-structural inflation and afterward the premium cap.



3.3.9.1 Non-structural inflation

Assume that the following values are known for $j \ge 2$ by PG (omitting PG in the notation) and without other contract group (CG) characteristic or gender distinction:

- κ_i^{PI} for the expected rate in premium increase of year *j* due non-structural inflation
- κ_i^{LI} for the expected rate in benefits increase of year *j* due non-structural inflation
- κ_i^{KI} for the expected rate in costs increase of year *j* due non-structural inflation

The above is not defined for j = 1 because the CY inflation is already included in estimations for CY; we set $\kappa_1^{PI} = \kappa_1^{LI} = \kappa_1^{KI} = 0$.

Then one can compute by PG $\gamma_j^{PI} = \prod_{\ell=2}^j (1 + \kappa_j^{PI}), \ \gamma_j^{LI} = \prod_{\ell=2}^j (1 + \kappa_j^{LI}) \text{ and } \gamma_j^{KI} = \prod_{\ell=2}^j (1 + \kappa_j^{KI}).$

For a particular *CG* and g, $\gamma_i^{PI,CG,g}$ is set the value γ_i^{PI} of the PG corresponding to *CG*.

The usual praxis is to offset the non-structural inflation in benefits by premium adaptations. The approach assumed in the standard model health for reflecting the parallelism in premiums and benefits increase is to set simultaneously for each future year $j \ge 2$ $\kappa_j^{PI} = \kappa_j^{LI} = \kappa_j^{KI} = 0$, i.e. $\gamma_j^{PI} = \gamma_j^{LI} = \gamma_j^{KI} = 1$. In other words, ignore the non-structural inflation in the calculation of the LZV.

3.3.9.2 Premium cap

We present formulae for the premium-cap factor $\gamma_j^{PC} \in (0,1]$ that possibly reduces the future expected premium of year *j*.

We use premium cap groups (PC-G), which are groups of CGs without gender distinction, at least the PG granularity, at most the CG granularity. An example of PC-Gs is the combination of PG and tarifftype with possibly the alternative method for premium cap (minimal combined ratio taken over all treatment years) applied only to PC-Gs related to entry age tariffs, see below.

The idea of premium cap is to set two parameters by PC-G (omitting PC-G in the notation): the minimal combined ratio CR_{min} , e.g. $CR_{min} = 90\%$ and the future year from which it applies $j_{min} \ge 2$, e.g. $j_{min} = 6$. Further, whether the minimal combined ratio is taken on yearly basis or for all treatment years together.

For the case where the minimal combined ratio is taken by year, the premium cap factor γ_j^{PC} is defined as follows, by PC-G:

For $j < j_{min}$, $\gamma_i^{PC} = 1$ else

$$\gamma_j^{PC} = \frac{CR_j}{CR_j^{PC}}$$



where

$$CR_{j} = \frac{\sum_{CG \in PCG} \sum_{g} \sum_{i \ge 1} \{ E(\bar{B}_{i,j}^{CG,g}) \cdot E(L_{i,j}^{CG,g}) + E(\bar{B}_{i,j}^{CG,g}) \cdot E(K_{i,j}^{CG,g}) \}}{\sum_{CG \in PCG} \sum_{g} \sum_{i \ge 1} \{ E(\bar{B}_{i,j}^{CG,g}) \cdot \check{E}(P_{i,j}^{CG,g}) \}}$$

with $\check{E}(P_{i,j}^{CG,g})$ defined by the expectation before application of premium cap, i.e. with $\gamma_j^P = \gamma_j^{PI}$ in the definition of $E(P_{i,j}^{CG,g})$

and

$$CR_{j}^{PC} = \max\left(CR_{j}, CR_{min}\right)$$

Alternatively, the minimal combined ratio can be taken over all treatment years as follows, by PC-G:

For $j < j_{min}$, $\gamma_j^{PC} = 1$ else $\gamma_j^{PC} = \bar{\gamma}_{j_{min}}$ where

$$\bar{\gamma}_{j_{min}} = \frac{\overline{CR}_{j_{min}}}{\overline{CR}_{j_{min}}^{PC}}$$

where

$$\overline{CR}_{j_{min}} = \frac{\sum_{CG \in PCG} \sum_{g} \sum_{j \ge j_{min}} \sum_{i \ge 1} \{ E(\overline{B}_{i,j}^{CG,g}) \cdot E(L_{i,j}^{CG,g}) + E(\overline{B}_{i,j}^{CG,g}) \cdot E(K_{i,j}^{CG,g}) \}}{\sum_{CG \in PCG} \sum_{g} \sum_{j \ge j_{min}} \sum_{i \ge 1} \{ E(\overline{B}_{i,j}^{CG,g}) \cdot \check{E}(P_{i,j}^{CG,g}) \}}$$

with $\check{E}(P_{i,j}^{CG,g})$ defined by the expectation before application of premium cap, i.e. with $\gamma_j^P = \gamma_j^{PI}$ in the definition of $E(P_{i,j}^{CG,g})$

and

$$\overline{CR}_{j_{min}}^{PC} = \max\left(\overline{CR}_{j_{min}}, CR_{min}\right)$$

The alternative (minimal combined ratio taken over all treatment years) may be useful for entry age tariffs in the sense that using high premiums at the beginning of the contract for financing future benefits is implicitly taken into account. Nonetheless it has the weakness of non-taking into account the discount effects for different treatment years in the sum over them, and even by redefining this alternative



with discount factors, it may provide not very stable premium cap factors over time. The alternative should be used with caution. Both formulae for the premium cap are implemented (see the R-function CapFators.f at Section 4.3.4).

It is possible that one PC-G uses the first approach and another the alternative. For a particular *CG* and *g*, $\gamma_i^{PC,CG,g}$ is set the value γ_i^{PC} of the PG corresponding to *CG*.

In principle γ_j^{PC} is calculated with and after γ_j^{PI} , γ_j^{LI} and γ_j^{KI} . Altogether, the formula for LZV is used with

$$\begin{split} \gamma_j^P &= \gamma_j^{PC} \cdot \gamma_j^{PI} \\ \gamma_j^L &= \gamma_j^{LI} \\ \gamma_i^K &= \gamma_i^{KI} \end{split}$$

For j = 1, γ_1^P , γ_1^L and γ_1^K do not appear in the formula for LZV and are not defined. For sensitivity analyses, e.g. to see what would be the impact of another estimation of CY inflation than the one used in estimations for CY, or what would outcome with a slightly higher CY premium, it is possible to define sensitivity factors γ_1^P , γ_1^L and γ_1^K (by default equal to one) and to replace $E(P_{i,1})$, $E(L_{i,1})$ and $E(K_{i,1})$ by respectively $\gamma_1^P \cdot E(P_{i,1})$, $\gamma_1^P \cdot E(L_{i,1})$ and $\gamma_1^P \cdot E(K_{i,1})$.

4 Documentation of the LZV-Tool

4.1 User guide

4.1.1 Installation

Step 1:

Please visit this link to install the software programs R and RStudio

https://posit.co/download/rstudio-desktop/

and follow the instructions or contact your IT department.

Step 2:

Use this link to install the software program Rtools

https://cran.r-project.org/bin/windows/Rtools/

Step 3:



Start RStudio. Select "Tools > Install Packages...", a dialog box will open:

Install Packages	
Install from:	? Configuring Repositories
Repository (FINMA_Repository)	~
Packages (separate multiple with spac	e or comma):
data.table	
Install to Library:	
C:/Users/f11589/RLibrary/R/win-libra	ry/4.3 [Default]
Install dependencies	
(Install Cancel

Select Repository (CRAN), enter one of the desired R packages (e.g. data.table) and click on "Install" (the screenshot shows the dialog box for data.table in the FINMA_Repository).

Do this step for the packages data.table, readxl, openxlsx, shiny, shinydashboard.

Step 4:

Unpack the archive *LZVTool010tar.zip* and save the R-package *LZVTool_0.1.0.tar.gz* on your local drive. Then select "Tools > Install Packages..." and "Install from: Package Archive". Click on "Browse..." to select the *LZVTool_0.1.0.tar.gz* archive.

Install Packages
Install from: Package Archive File (.zip; .tar.gz)
Package archive:
C:/Users/f11589/Documents/LZVTool_0.1.0.tai Browse
Install to Library: C:/Users/f11589/RLibrary/R/win-library/4.3 [Default]
Install Cancel

This step completes the installation.

4.1.2 Using the LZV-Tool package with a GUI

After the program has been installed, the tool can be started using the following command in Rstudio:

library(LZVTool)



After the tool has been loaded into RStudio, the dashboard can be started with the following command:

IzvDashboard()

This command opens the following window:

C:/Users/f11589/Documents/LZVTool/inst/shiny - Shiny - □							
http://127.0.0.1:5008	😏 Publish 👻						
	LZV Dashboard	·					
≡							
Input SS	T-Health-Template						
Browse	No template selected						
	Calculate LZV						
		•					

The corresponding SST-Health-Template can be selected here with "Browse...". Then click on "Calculate LZV" to run the program and after the calculation a path can be selected to save the results.

It is possible to call up the latest changes to the program with the following command:

lzvNews()

4.2 Description of the main functions

In this section, we describe the main functions from the LZV-Tool package which can be used for more focused or advanced analysis.



4.2.1 getModelInput

This is the function that imports the data from the Excel template in a structured manner.

Input:

• Path: the path to the adapted SST-Health-Template.

Output:

• An object of class "sstHealthTemplate" which contains the input Excel data.

4.2.2 getHealthDataModel

This is the function that transforms the Excel inputs from the adapted SST-Health-Template into R objects.

Input:

• modelInputs: an object of class "sstHealthTemplate" as obtained from the "getModelInput" function.

Output:

- An object of class "sstHealthDataModel" which contains the input data as well-defined R objects.
 - cgNames: a list of unique identifiers for each contract group.
 - cgData: a list containing, for each contract group, the premiums, the benefits and the nunber of contracts
 - costFactors: a list of cost factors for each product group.
 - standardParam: a list containing the prescribed parameters (such as mortality, etc.)
 - alphaOne: the parameter in the definition of the mean headcount.
 - PGinflation: the inflation assumption for each product group.
 - antiSelection: the factors per age class for the anti-selection scenario.
 - discountFactors: the discount factors.
 - shiftMagnitude: the table giving the up and down shifts for each risk factor used to compute the risk model.
 - shiftYearlyApplicability: the table indicating the years to which to apply the shifts for each risk factor used in the risk model.
 - caps: a list of parameters to compute the premium capping factors for each possible product group (c.f. the CapFactors.f function below).
 - config: a list of configuration parameters for each contract group.
 - companyName: the company name as indicated in the "Intro_SM_Health" sheet.
 - currentYear: the year as indicated in the "Intro_SM_Health" sheet.



- suffix: the suffix for the file name as indicated in the "Intro_SM_Health" sheet.
- suffixTitle: the suffix for the title as indicated in the "Intro_SM_Health" sheet.

4.2.3 getOutputData

This function computes the main results.

Input:

 healthModel: an object of class "sstHealthDataModel" as obtained from the "getHealthDataModel" function.

Output:

- A list of class "sstHealthData" containing the following data
 - companyName: see the description of getHealthDataModel.
 - currentYear: see the description of getHealthDataModel.
 - suffix: see the description of getHealthDataModel.
 - suffixName: see the description of getHealthDataModel.
 - totalCashflows: the aggregated undiscounted and capped premium, benefits and cost cash flows for all products.
 - CGCashflows: the undiscounted and capped cash flow for each contract group.
 - PGCashflows: the undiscounted and uncapped total cash flow for each product group.
 - PGCashflowsCapped: the undiscounted and capped total cash flow for each product group.
 - numberOfContracts: the evolution of the number of contracts for each product group.
 - antiSelectionCF: the total undiscounted and capped cash flow after the application of the antiselection scenario for each product group.
 - shiftCashflows: the total undiscounted and capped cash flows for each shift in the risk factor used in the risk model.
 - discountRates: see the description of getHealthDataModel.

4.3 Description of the basic computation functions

The LZV-Tool package follows a functional approach in which basic functions are defined, which are then called in more complex functions to obtain the relevant results. The basic functions thus serve as the "heart" of the program and perform the most fundamental computations. They are implemented in the file *LZVCalculation.R*.

For each of these basic functions, we describe the input parameters and the output values.



4.3.1 QUDev.f

This function develops an initial quantity by unit (for example premiums by contract) into a matrix following the method described in Section 3.3.7.2. This function allows to set a threshold age after which the quantity is constant, which would be the case, e.g., for entry age tariffs or old age liabilities.

Input:

- QUInit: vector of length MaxAge + 1 with the QUs of the current year CY.
- ThresholdAge = 91: real age from which QU is constant. Typical applications are for entry age tariffs or for benefits for old people.
- MaxAge = 110: the oldest age considered in the calculation. Also defines the size of output matrix.

Output:

• QU: matrix with quantity by unit with indices (i, j), where i = CYage + 1 and j = 1 for CY, j = 2 for CY + 1, etc.

4.3.2 FactDev.f

This function provides the cumulative development factors for a quantity, e.g., survival probability over the years.

Input:

- MortalityInit: vector of the yearly mortality by age in the current year. Has to have length MaxAge + 1 and last element has to be 1.
- LapsesInit: vector of the yearly lapse probability in the current year. Has to have length MaxAge + 1.
- MaxAge = 110: highest age considered.
- mortShiftVec = 1: vector with shifts year to year by which the mortality is multiplied. (Used for risk model.)
- lapsesShiftVec = 1: vector with shifts which are multiplied to lapses. (Used for risk model.)

Output:

• FC: quadratic Matrix with the survival probabilities with indices (i, j), where i = CYage + 1 and j = 1 for survival CY, j = 2 for CY + 1, etc.

4.3.3 BesDev.f

This function provides the development of a number of units over the years, typically the number of contracts.



Input:

- BesInit: vector with the number of contracts at t = 0. Has to be size MaxAge + 1.
- MortalityInit: vector of the yearly mortality by age in the current year. Has to have length MaxAge + 1 and last element has to be 1.
- FC: matrix with probability of a contract staying active to a point in the future. Has to be quadratic and have MaxAge + 1 columns.
- alphaOne: value between 0 and 1 used for computing the mean number of contracts.
- BesDevMax = 50: year after which all exposure is set to 0.
- MaxAge = 110: maximum Age considered in calculation.

Output:

• Bes: matrix of volumes of units where rows represent the different age classes, and the columns represent the years in the future.

4.3.4 CapFactors.f

This function computes the multiplicative factors for capping premiums.

Input:

- PT_j: vector with total premiums per year with length MaxAge + 1.
- LT_j: vector with total benefits per year with length MaxAge + 1.
- KT_j: vector with total costs per year with length MaxAge + 1.
- FromDev = 6: integer year from when the cap is in effect. Can not be lower than 2.
- CRThres = 0.9: threshold lowest combined ratio, i.e. (benefits + costs) / premiums.
- yearlyCR = TRUE: logical, if True, the Cap has to hold for every single year. If FALSE, the cap only applies for the full duration from FromDev to MaxAge + 1.
- MaxAge = 110: the highest considered age. Also defines the length of the vectors.

Output:

• CapFactors: vector of factors with length MaxAge + 1. 1, if the cap condition is already fulfilled, a factor smaller than 1 otherwise. This factors vector can be multiplied to premiums to obtain the caped premium.